Particle-in-Cell Simulations of Anomalous Transport in a Penning Discharge

IEPC-2017-523

Presented at the 35th International Electric Propulsion Conference
Georgia Institute of Technology • Atlanta, Georgia • USA
October 8 – 12, 2017

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Abstract: Large scale, low frequency, rotating structures, often known as "spokes" have been observed in a wide range of devices, including Hall thrusters, Magnetrons and Penning discharges. The presence of these structures is often associated with anomalous transport of electrons across the applied magnetic field, resulting in reduced device efficiency. We self-consistently simulate a Penning discharge cross-section via a 2D-3V Particle-in-Cell Monte-Carlo-Collision (PIC-MCC) code. Improvements to the code and problem scaling allow us to simulate up to 100 µs of plasma time on 24-28 processors over 2-4 days. A rotating spoke was observed and found to correlate strongly to anomalous transport, with Hall parameters two to three orders of magnitude larger than the classical value. Evidence points towards a collective flute mode being responsible for spoke formation.

Nomenclature

E = Electric field (electric field vector if boldfaced)
B = Magnetic field (magnetic field vector if boldfaced)
m = Azimuthal mode number
M = Ion mass
n = Plasma density
r = Radius
R = Trap radius
β_{eff} = Effective Hall parameter
e = Charge of electron
σ = Conductivity
I = Current

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t = Thickness (depth)
T_i = Temperature of species i

I. Introduction

Hall plasmas, in which electrons are magnetized and ions are unmagnetized, exhibit a wide range of plasma instabilities. For systems which exhibit closed drifting of electrons in the $E \times B$ direction, a prevalent instability is that of the so-called plasma "spoke". This instability is most readily observable as a large scale, low frequency, low mode number, fluctuation in plasma density, propagating in the $E \times B$ direction. Devices which exhibit such topology and within which the spoke has been observed include Hall thrusters[1] and Magnetrons[2] among others. These devices exhibit cylindrical geometry, with closed electron drifts and spoke propagation occurring in the azimuthal direction. Interest in the plasma spoke within such devices has primarily been due to the suspected role that they play in anomalous transport of electrons across the applied magnetic field, a source of reduced efficiency.

The Penning discharge is a plasma device, ideal for studying some of the most important features of Hall plasmas, including the formation of coherent structures and anomalous transport. Plasma is generated at the center of the trap by an ionizing beam of electrons. Electron motion in the radial direction is inhibited by an applied uniform axial magnetic field. Ions are weakly magnetized and therefore significantly more mobile, giving rise to an ambipolar radial electric field. The applied field and ambipolar electric field result in electrons undergoing $E \times B$ drifts in the azimuthal direction, similar that seen in Hall thrusters and Magnetrons. The primary advantage of this configuration is increased access for plasma diagnostics, such as probes and line of sight optical diagnostics. Furthermore, the physics is somewhat simplified by the lack of a magnetic field gradient, removing one source of energy for driving instabilities.

Early experimental investigation of the rotating spoke within a Penning discharge[3], revealed the presence of an $m = 1$ flute instability with a frequency between 25-50 kHz. The presence of this instability was found to correlate strongly with enhanced electron transport. Modern investigations include those within the Mistral device at the University of Marseille[4], similarly showed that a low mode number structure was correlated with anomalous transport[5]. Work done at the Princeton Plasma Physics Laboratory (PPPL)[6] revealed that the spoke frequency within a Penning discharge scales as $(B/M)^{1/2}$.

Despite the prevalence of spoke like structures within Hall plasmas, a theoretical understanding of their formation has remained elusive. This is most likely due to the highly non-linear, turbulent and global nature of the mode. Historically it was considered that the spoke formed as an ionization wave[1,7] based on the concept of the Critical Ionization Velocity (CIV)[8-9]. This theory has remained prevalent within Magnetron literature, since the neutral pressures within such devices may be sufficient to sustain such a wave[10]. Within Hall thrusters and Penning discharges however, the gas pressure is significantly lower and alternative theories suggest that the spoke may develop as a flute mode which saturates at large amplitude, or forms from an inverse cascade of higher wavenumber flute modes[5,11-12]. The Modified Simon-Hoh Instability (MSHI) is considered as the most likely contender responsible for these fluctuations[13]. The MSHI is similar to the Simon-Hoh instability[14-15], in that the instability is driven by charge separation due to differing drift velocities of ions and electrons. In Hall plasmas, the ions are weakly magnetized and slower ion drift speeds are the result of finite Larmour radius effects. Instability occurs when the electric field and density gradient are aligned $\nabla n \cdot \boldsymbol{E} > 0$, a criterion which is satisfied in the Penning discharge.

Since 2000, there has been considerable computational efforts towards modeling of Hall plasmas, particularly for reproducing the effects of anomalous transport[16]. In fluid and hybrid simulations, electron transport is incorporated by a model, with parameters tuned from experiments. This calls into question the range of validity of such models. Kinetic methods, such as Particle-in-Cell techniques combined with Monte-Carlo Collisions (PIC-MCC) avoid this issue as they evolve the plasma self consistently, with the only modeling being incorporated via the Monte-Carlo-Collision (MCC) algorithms, which are generally considered to be well understood and accurate over a wide range of collisional conditions. The challenge with PIC-MCC comes with the significant computational overhead required to capture the vastly multi-scale physics observed within devices such as Hall thrusters. This challenge is often alleviated by scaling the problem in some way, whether it be through device size[17], plasma density or relative permittivity[18].

Improvements in modern computing have led to a number of recent efforts to model Hall plasmas in two-dimensions[19-20], and even entire three-dimensional devices[17,21-23]. However, few simulations have focused on the modeling of a Penning discharge. If it is assumed that the transverse and longitudinal dynamics are decoupled, the Penning discharge has the added advantage that it can be modeled by a two-dimensional system. This simplified geometry, and associated reduced computational cost, make it an ideal system in which to perform detailed studies of anomalous transport and formation of the plasma spoke. In this paper, we continue the work commenced in Refs. 24-25, modeling the formation of a rotating spoke and associated anomalous transport within a Penning discharge.

II. Simulation Model

In our simulations, we utilize the electrostatic module of the PIC-MCC program Large-Scale Plasma (LSP). In combination with its extensive history of validation and use within the literature[25-26], LSP features a robust and intuitive geometry builder, a wide range of particle creation methods, and collision algorithms. LSP was designed to be coupled with the Portable Extensible Toolkit for Scientific Computation (PETSc)[27] for access to state-of-the-art linear algebra routines. We modified LSP for compatibility with the latest version of PETSc (version 3.7) and coupled it with the LU factorization package SuperLU[28] for a fast, accurate and efficient solution of Poisson's equation. Particle pushing was performed explicitly by the standard Boris algorithm[29] and interpolation by either the nearest cell or cloud-in-cell methods. MCC algorithms included elastic and ionizing collisions between electrons and neutrals only.

Our starting point for the model of the Penning discharge was based off that of the device at PPPL[6]. The discharge features an RF plasma cathode which injects electrons along the axial magnetic field into either an Argon or Xenon background gas. The plasma is surrounded with a grounded cylindrical metal casing, acting as the anode. The ends of the cylinder are dielectric, preventing the loss of particles in the axial direction. The device geometry and simulation geometry are shown in Fig. 1.

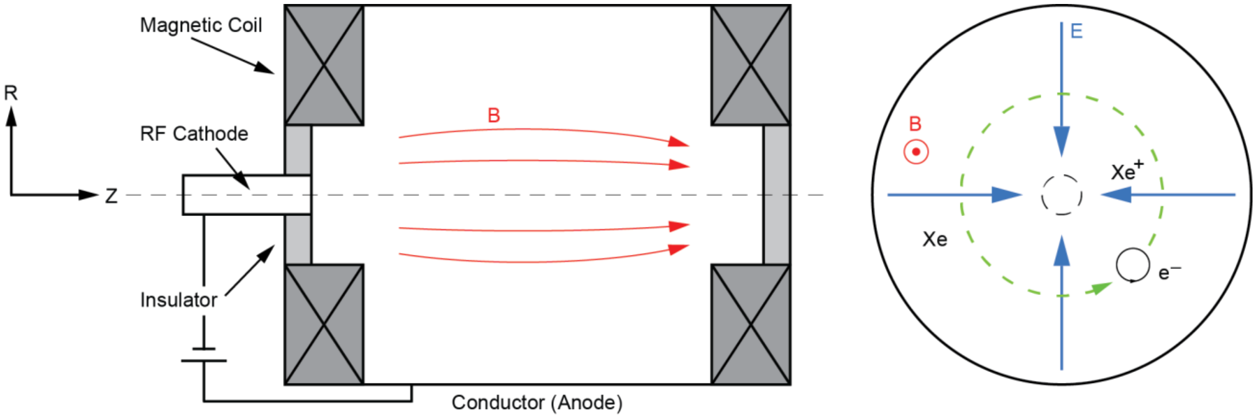

Figure 1: a) Experimental setup of the Penning discharge at PPPL [2015 Raitses] in r-z geometry. Magnetic field lines are indicated in red. b) cross-section of the Penning discharge, representative of the simulation domain in r-θ geometry. Blue lines indicate the radial ambipolar electric field and green-dashed lines represent the direction of electron $E \times B$ drift. Xenon neutrals and ions are unmagnetized.

We model the Penning discharge in 2D-3V geometry on a regular uniform Cartesian grid with a constant and uniform magnetic field applied perpendicular to the simulation domain (B = 100 gauss). The grid was chosen to be Cartesian, so as to avoid numerical instabilities associated with the grid singularity at zero radius in cylindrical geometry. The cathode was modeled as a uniform 15 eV beam of electrons injected perpendicular to the simulation domain with a current of 250 mA. The anode was modeled as a cylindrical stair-stepping boundary with a constant Dirichlet potential. Particles which impact the anode are absorbed and lost from the simulation. Both injected electrons, and electrons and ions formed from ionization were allowed to evolve, however the neutrals were present as a fixed background, with constant pressure (1 mTorr) and temperature (293 K).

A major goal of this research was to develop a stable simulation which could model real sized devices over simulation times of several days. In order to achieve this, initial models were scaled to improve simulation time, with the plan being that these restrictions could slowly be removed with improvements to the code. With this in mind, the simulated domain was reduced in diameter from 10cm to 5cm. Furthermore, we simulated a Xenon gas with mass reduced to that of Helium. Finally, we scaled the relative permittivity of the simulation up to $\varepsilon_r = 400$, reducing constraints on resolving Debye length (and consequentially the Courant condition). It was chosen to scale relative permittivity, since the dispersion relationship of the MSHI is seen to be independent of this parameter. This resulted in a grid size of 250 x 250 cells with cell side length 0.2 mm. All plasma length scales and time scales were properly resolved and the Courant condition was satisfied for evolving particles. These simplifications allowed us to simulate 100 microseconds of plasma time over a period of 2-4 days. Simulation were run with 28 cores on the Princeton University supercomputer Perseus[30] and 24 cores on the Department of Energy's National Energy Research Scientific Computing Edison supercomputer[31].

III. Results and Discussion

A. Observation of the Rotating Spoke and Anomalous Transport

There is no plasma present at the start of the simulation. Electrons are injected into the uniform background of neutral plasma resulting in ionization and the formation of additional electrons and of ions. The simulation is therefore initially non-neutral and a rapid expansion of the particles is observed as the electrons attempt to escape the space charge present within the center of the simulation.

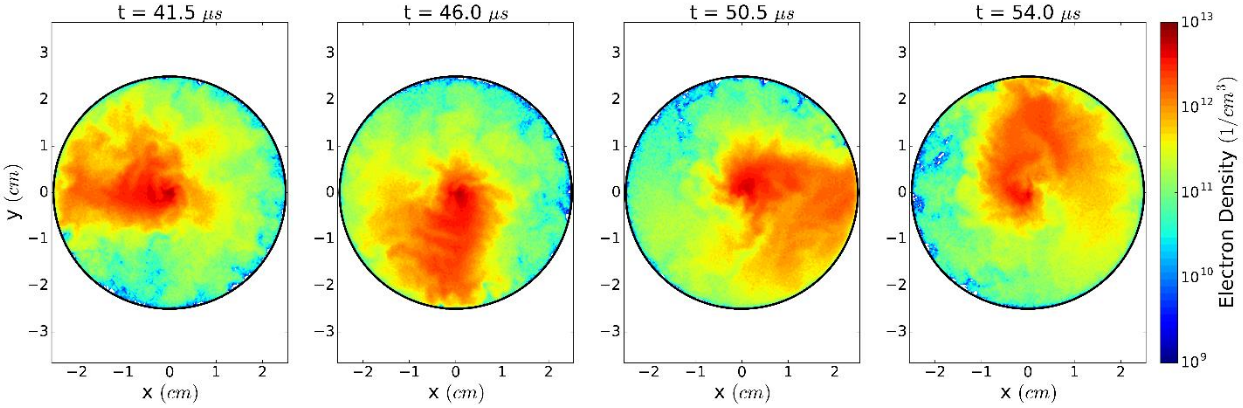

Figure 2: a) Plots of electron density (logarithmic scale) at 41.5 µs, 46.0 µs, 50.5 µs, and 54.0 µs after commencement of the simulation. The spoke is clearly present as a rotating density perturbation with a frequency of 64 kHz.

As the plasma evolves to a quasi-neutral state, high mode-number azimuthal waves begin to propagate perpendicular to the density gradient. At around 15-20 µs after commencement of the simulation, these waves coalesce into a single structure (see Fig. 2), rotating in the $E \times B$ direction. This structure persists throughout the remainder of the simulation (up to 100 µs). The structure appears to be rather chaotic, turbulent, and clearly consists of radial as well as azimuthal wavenumbers as evidenced by the "shedding" of density during rotation. The frequency of the mode is measured as 64 kHz. Waves are observed forming and propagating along the density gradient iso-contours of the spoke itself, perhaps acting as a source for driving the instability.

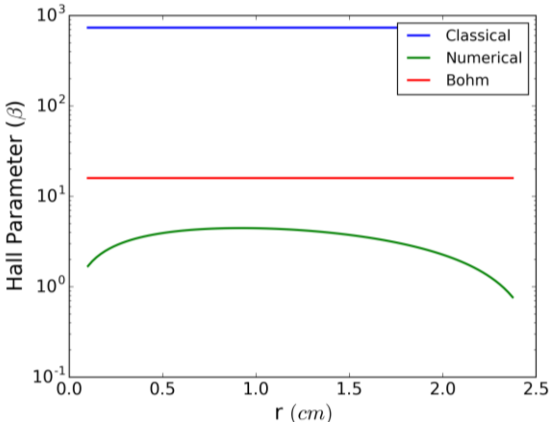

Figure 3: Hall parameter (β) against radius for classical theory, Bohm theory and as calculated from the numerical experiment within the Penning discharge.

For a conducting cylinder with a Dirac delta central source of ions and electrons, and assuming a uniform diffusion constant with radius, the density profile takes the form $n(r) = n_0 \ln(r/R)$. Where n_0 is some normalizing density and R is the trap radius. The density cross-section of the simulation was averaged over several periods of the spoke structure and was found to have reasonable agreement with the above analytic relationship, with $n_0 = 10^{12}\ 1/cm^3$ as a fitting parameter. This density profile was used in calculations given below.

The electric field was averaged over several periods of the spoke. Outside of the beam region and anode sheath the field was found to be approximately constant with strength, $E = 115\ V/m$. Assuming a constant current ($I = 250\ mA$) flowing outwards at each radial location the radial dependence of the electron conductivity $\sigma(r)$ can be calculated from Ohm's law. Combined with knowledge of the plasma density profile $n(r)$ and applied field strength B, an effective Hall parameter β_{eff} could be calculated as:

$$\beta_{eff} = \frac{n(r)e}{B\sigma(r)} = \frac{n_0 e I}{2\pi r t(E + T_e/(er\ln(R/r)))B} \ln\left(\frac{r}{R}\right) \tag{1}$$

Where T_e is the average electron temperature, e is the elementary charge and t is the depth of the simulation in the axial direction (LSP defaults to 1cm).

A plot of the numerical Hall parameter against the classical Hall parameter (ν/ω_c where ν is the collision frequency calculated for 5.0 eV electron and a Xenon gas pressure of 1 mTorr) and the Bohm Hall parameter (for γ = 1/16) are plotted in Fig 3. It is seen that the Hall parameter for the numerical experiment exceeds the Bohm value by a factor of 3-30 and greatly exceeds the classical value by two to three orders of magnitude. This demonstrates that the electron transport is highly anomalous, consistent with the literature.

To measure the correlation between spoke rotation and radial current, a numerical probe was placed at $(x, y) = (0, -2.45)$. Figure 4 shows that fluctuations in radial electron current correlate strongly with the increased electron density associated with the passage of the rotating spoke. This indicates that transport is strongly enhanced within the spoke.

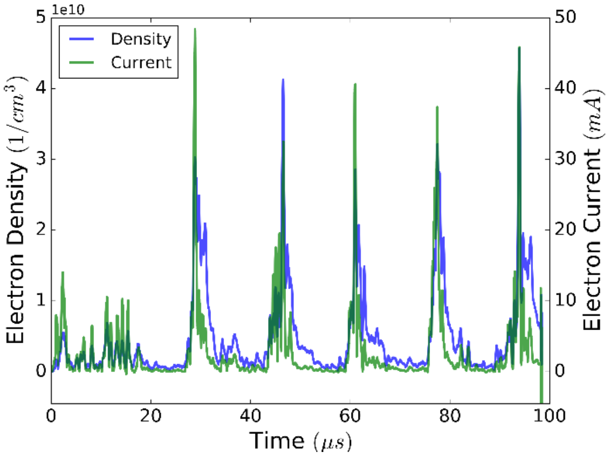

Figure 4: Electron density and radial electron current against time at $(x, y) = (0, -2.45)$.

B. Particle Trajectories

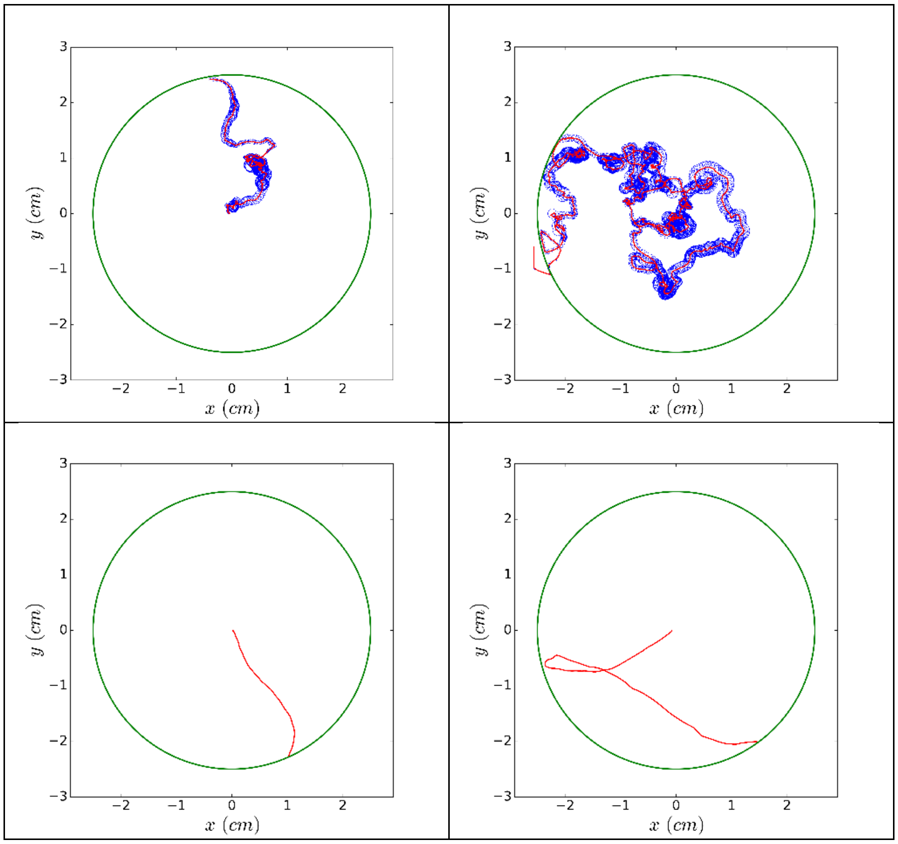

Figure 5: Plots of particle trajectories. a) and b) demonstrate typical electron trajectories, blue lines are the true electron paths and red lines are the gyro-center paths. c) and d) demonstrate typical ion trajectories.

Figure 5. demonstrates typical ion and electron trajectories within the Penning discharge. Electron and ion lifetimes within the trap are around 5 μs and 15 μs respectively. Although these times vary wildly for individual particles. It is clear from Figs. 5a) and 5b) that the electrons do not undergo any clearly defined circular orbits due to either $E \times B$ or diamagnetic effects. Their paths are chaotic and seemingly random, perhaps indicative of a highly turbulent system.

Figures 5c) and 5d) clearly demonstrate that the ions are unmagnetized. Most ions directly leave the simulation, as in Figure 5c), indicating poor confinement by electrostatic forces. Some ions are reflected near the wall of the device, as in Fig 5c), however, this generally occurs only once or twice prior to particle loss.

C. Rotating Spoke without Ionization

As discussed in Section I, there remains the outstanding question on the fundamental mechanism responsible for the formation of the spoke. Taking advantage of our ability to control the physics within our simulation, it was chosen to model the Penning discharge without the use of ionization, or indeed any form of collisions, such that the physics was modeled by purely collective effects. The electron beam was replaced with an isotropic source of thermal electron ($T_e = 5\, eV$) with the same current of 250 mA and to maintain quasi-neutrality an isotropic source of ions ($T_i \sim 0.025\, eV$) of 100 mA was also injected at the center of the trap.

Self-organization into a spoke like structure occurred over a longer time period than within the collisional case. However as show in Fig. 6, a spoke with a frequency of 59 kHz became clearly observable after around 30 μs of simulation time.

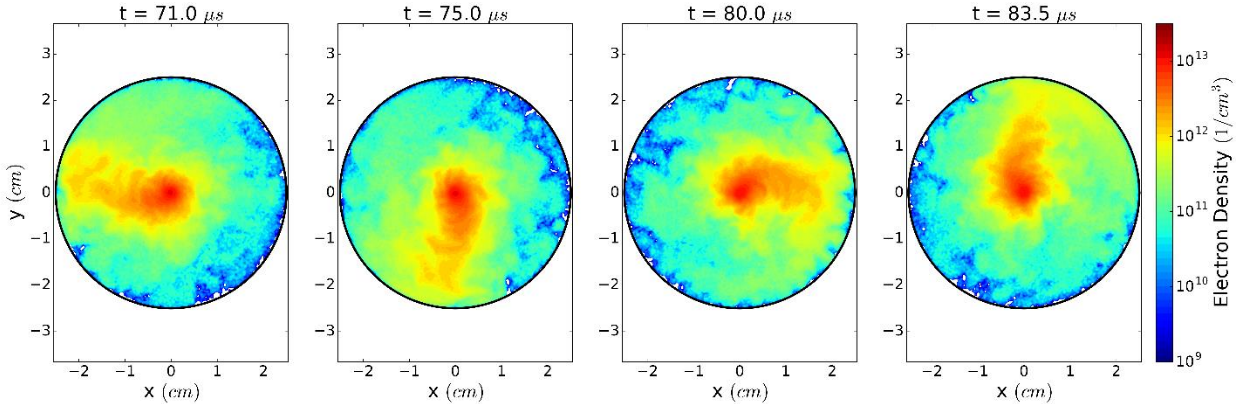

Figure 6: Plots of electron density (logarithmic scale) within a collisionless Penning discharge simulation at 71.0 μs, 75.0 μs, 80.0 μs, and 83.5 μs after commencement of the simulation. The spoke is clearly present as a rotating density perturbation with a frequency of 59 kHz.

This demonstrates that within the Penning discharge, a spoke instability can form without any collisional effects. Therefore, the fundamental mechanism responsible for the spoke formation within this device cannot be related to an ionization wave via the CIV concept. The similarity between the spoke frequency, as observed in Fig. 2 and Fig. 6 most likely indicate that even for low pressures, the spoke within a Penning discharge is not due to an ionization wave. These conclusions however are not necessarily applicable to other Hall plasma devices, where the operating conditions may differ significantly.

IV. Conclusion

A Penning discharge was modeled in two-dimensions via the self-consistent PIC-MCC kinetic method. A spoke like structure with a frequency of 64 kHz, was observed to rotate in the $E \times B$ direction. By modifying the simulation physics to exclude particle collisions, it was found that the spoke was not an ionization wave. It is suggested that the Modified-Simon-Hoh Instability, a flute mode, may play some role in the spoke formation.

The transport of electrons within the trap was anomalously high, with a Hall parameter two to three orders of magnitude larger than the classical value. Radial electron current was found to correlate strongly with the location of the spoke, indicating that anomalous transport is occurring along the spoke structure.

These simulations were conducted on a time scale appropriate for in depth parametric scans and a detailed analysis of the spoke physics. Future work will continue to focus on realistic device geometries, as well as the fundamental physics responsible for the formation of the rotating spoke.

Acknowledgments

This work was supported by the Air Force Office of Scientific Research.

This work was supported by the Princeton University Program in Plasma Science and Technology.

This research used resources of the National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

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