

# Effect of Low-frequency Oscillation on Performance of Hall Thrusters

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**Abstract:** In this paper, we study the effect of low-frequency oscillation amplitude on the thrust and anode efficiency of a Hall thruster by varying the capacitance of the filter unit of the thruster, and preparing different low-frequency oscillation amplitudes of the discharge current at constant steady discharge parameters. To be precise, the variation characteristics of the ion current, propellant utilization and divergence angle of the plume at different low-frequency oscillation amplitudes are measured, and then the underlying mechanism of low-frequency oscillation for the performance parameters of thrusters is analyzed. The findings demonstrate that at low voltage and small flow, with an increase in the low-frequency oscillation amplitude, the propellant utilization increase, and the divergence angle of the plume increases from 9.8° to a maximum of about 13.8°. The thrust increases with an increase in the low-frequency oscillation amplitude, and the anode efficiency increases as well. However, at high voltage and large flow, although an increase in the low-frequency oscillation amplitude slightly increases the number of ions in the channel, the divergence angle of the plume increases from 11.7° to a maximum of about 19.3°, and the loss of plume divergence increases; therefore, the thrust decreases with an increase in the low-frequency oscillation amplitude, and the anode efficiency decreases.

## I. Introduction

Hall thrusters have been widely applied in attitude control, position keeping, orbital transfer of satellites, and other space propulsion missions due to the advantages of their simple structure, moderate specific impulse, high

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efficiency, etc.<sup>[1,2]</sup>. During the discharge of a Hall thruster, a 20-40 kHz low-frequency oscillation occurs<sup>[3-6]</sup>. The variation characteristic in this low-frequency oscillation is closely related to the ionization and acceleration of the neutral gas in the discharge channel. In addition, the ionization and acceleration of the neutral gas determine the thruster performance. Therefore, it is necessary to study the relationship between the low-frequency oscillation amplitudes and thruster performance.

In the past few decades, researchers have conducted several studies on low-frequency oscillation. For example, Yamamoto et al.<sup>[7]</sup> studied the influence of magnetic field strength on the low-frequency oscillation of Hall thrusters; they found that with the increase in magnetic field strength, the discharge current amplitude decreases initially and later increases, while the thrust slightly decreases, and the thruster efficiency is as low as ~20% at a magnetic field strength larger than 270 G. Lobbia & Gallimore et al.<sup>[8]</sup> experimentally studied the two-dimensional variation of low-frequency oscillation and plume parameter of thrusters, with discharge voltages ranging within 200-400 V. Their experimental result shows that with an increase in the discharge voltage, the low-frequency oscillation intensifies in the range of 18-46 kHz, and the average ion velocity increases from 10.4 km/s to 17.4 km/s. Gascon & Dudeck et al.<sup>[9]</sup> experimentally investigated the effect of different wall materials on low-frequency oscillation and found that the discharge current amplitude fluctuates by more than 25% due to the discharge of different wall materials, and the thrust increases by ~10%. Moreover, both thrust and discharge efficiency dramatically increase with a gradual increase in the discharge voltage for different wall materials. Tahara et al.<sup>[10]</sup> investigated the effect of three types of channel wall materials—boron nitride (BN), boron nitride-silicon nitride (BNSiN), and boron nitride-aluminum nitride (BNAlN)—on the plasma characteristic and performance of the thruster. Their comparison shows that using the same magnetic field strength, the discharge current oscillation is the smallest when BN is used as the wall material, with a discharge voltage of 200 V and an anode mass flow rate of 2.0 mg/s. Moreover, both thrust and anode efficiency are the largest under these conditions. Zhang Xu et al.<sup>[11]</sup> added an azimuthal diversion rail to the thruster anode and studied its influence on the ionization rate in the channel. They found that the low-frequency oscillation amplitude of the discharge current of the thruster increases with a gradual increase in the mass flow rate upon using an azimuthal diversion rail, while both the thrust and anode efficiency of the thruster increase at low mass flow rate. Furthermore, the divergence angle of the plume decreases as expected. Garrigues et al.<sup>[12]</sup> studied the influence of different magnetic field configurations on the thruster performance by establishing a two-dimensional hybrid model. Their simulation result shows that all overall efficiency of the thrust and specific impulse of the thruster remain nearly unchanged in different magnetic field configurations. Tahara & Goto et al.<sup>[13]</sup> studied the variation characteristics of plasma characteristic and performance of a low power Hall thruster at different magnetic field strengths. They found that the discharge current amplitude initially drops, and then increases slowly with the gradual increase of the magnetic field strength from 100 G to 200 G. Meanwhile, the thrust remains nearly unchanged, the thruster efficiency increases initially and decreases later. With the gradual increase of the discharge voltage, the amplitude of discharge current increases linearly at first and remains unchanged for a certain range later, while both thrust and efficiency tend to rise as expected. Many related findings show that the influence of low-frequency oscillation on the performance of Hall thrusters is still unknown, because it is uncertain whether the thruster performance changes due to the change in the operating parameters or due to the discharge current oscillation, as some researchers have observed a significant change in the performance when the operating parameters of the thruster were varied in order to study the oscillation characteristics.

In this paper, a direct connection between the discharge current amplitude and the thruster performance is established by varying solely the capacitance of the filter unit of the Hall thruster and the effects of oscillation of the low-frequency discharge current on thrust and anode efficiency of thrusters at different operating points were studied. To be precise, the variation characteristics of the ion current, propellant utilization and divergence angle of the plume during oscillation were measured. Finally, we describe the mechanism of how the current oscillation influences the performance parameters of thrusters.

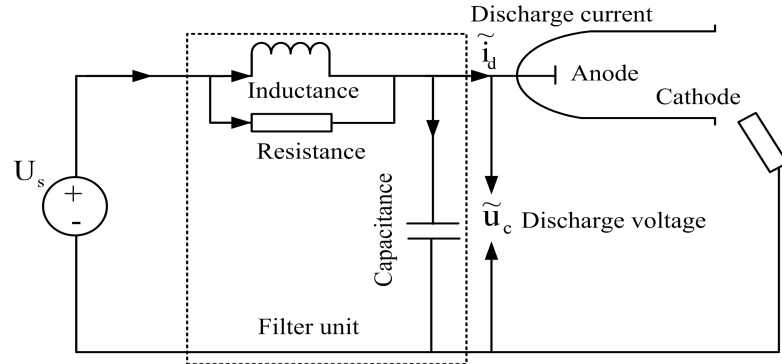
## II. Experimental Setup

The main apparatus used in the experiments includes a laboratory prototype Hall effect propeller called HEP 140, a set of vacuum systems, a thrust stand and a plume diagnose probe system.

### A. The HEP 140 Hall thruster and Vacuum facility

Our studies are carried out on an ATON-type Hall thruster. The diameter of the outer insulator of the experimental thruster is 140 mm and that of the inner insulator is 100 mm. The length of the discharge channel is 35 mm. A cold cathode was used to provide electrons and to neutralize ejected ions. All experimental trials were performed in a  $2 \times 5$  m cylindrical vacuum chamber, equipped with a coarse pump, two cryogenic pumps, and four

xenon pumps, each with nominal xenon pumping speed of 27 kl/s. Before operation, the back pressure of the vacuum was  $1.0 \times 10^{-4}$  Pa, which is corrected for xenon. The diagram of the measurement scheme is shown in Fig. 1. The discharge current was measured with a Yokogawa DL850 ScopeCorder instrument at the anode terminal.



**Figure 1 Equivalent Principle Diagram of Measured Operating Circuit of Hall Thruster**

### B. Thrust stand

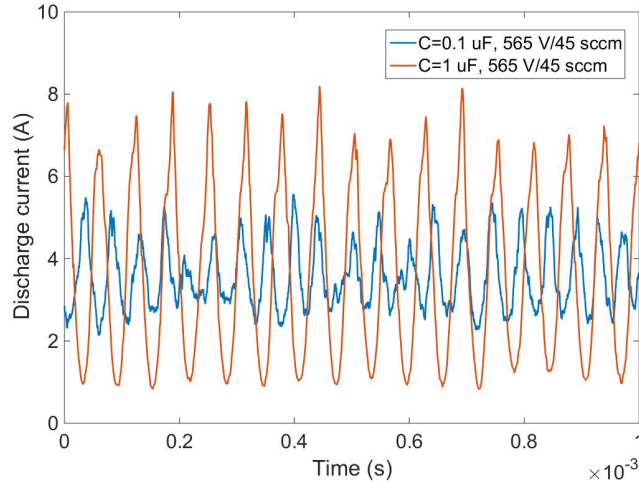
The microthrust was measured by a three wire torsion pendulum. By applying the principles of torque balance and optical lever amplification, the microthrust is transformed to a spot of large displacement. In order to measure thrust more accurately, when the discharge is off, the thrust should be calibrated by weights. Ambient temperature and other factors usually have a certain impact on the accuracy of the thrust measurement; therefore, a standard remote calibrating technique based on weights is applied to eliminate this problem. The thrust measuring resolution was  $\pm 0.3$  mN and the precision was about 0.5% at 100 mN and 2% at 20 mN. The schematic of the torsion pendulum can be found in Ref. [11].

### C. Faraday probe

Faraday probe system was used to measure the plasma parameters of the plume<sup>[8]</sup>. The Faraday probe was mounted on a rotation stage with a distance of  $R=25$  cm from the pivot to the thruster axis, with the ability of turning at angles up to  $\pm 90^\circ$ . The ion collecting area,  $A_p$ , of the Faraday probe was  $0.38$  cm<sup>2</sup>. Both the probe collector and guard ring were biased to  $-24$  V relative to the potential of the cathode, in order to achieve a saturation ion current. The total ion current and plume divergence half-angle can be calculated by using Eq.s (1) and (2) in [8], respectively. The detailed structure of this probe system can be seen in Hu Peng et al.<sup>[14]</sup>.

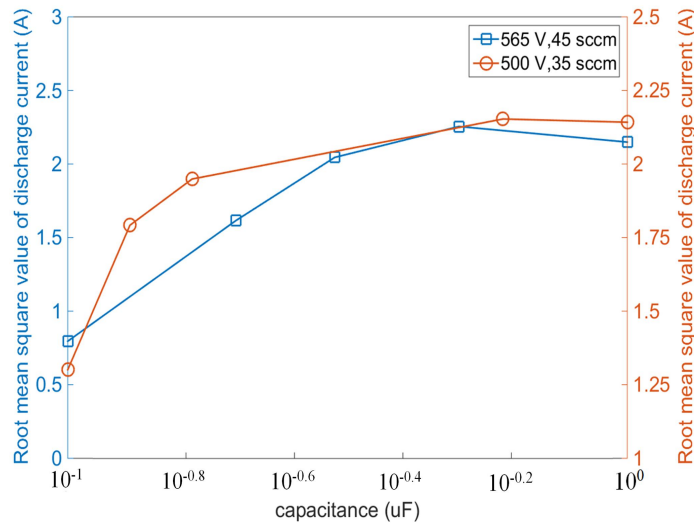
## III. Results

At constant discharge voltage, mass flow rate, magnetic field strength, and other steady state parameters, the capacitance of the filter unit is adjusted to achieve different discharge oscillation amplitudes (as shown in Fig. 2). At the discharge voltage of 565 V and the mass flow rate of 45 sccm, i.e., high voltage and large flow, or the discharge voltage of 500 V and the mass flow rate of 35 sccm, i.e., low voltage and small flow, the filter resistance and the inductance are kept at and , respectively. The capacitance is changed from to in order to obtain different discharge current amplitudes, while the thrust and anode efficiency of the thruster at different amplitudes are measured, as shown in Figs. 3, 4, and 5.



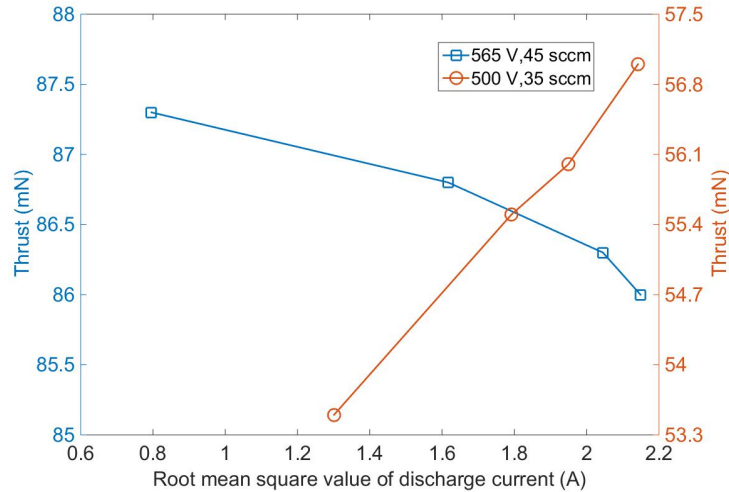
**Figure 2. Measurement results of low-frequency oscillating currents at different capacitance values**

Figure 3 shows that the discharge current amplitude initially increases rapidly and slows down subsequently, with the increase of capacitance at constant steady discharge parameters. The oscillation amplitude does not change as expected when the capacitance exceeds  $0.8 \mu\text{F}$ .

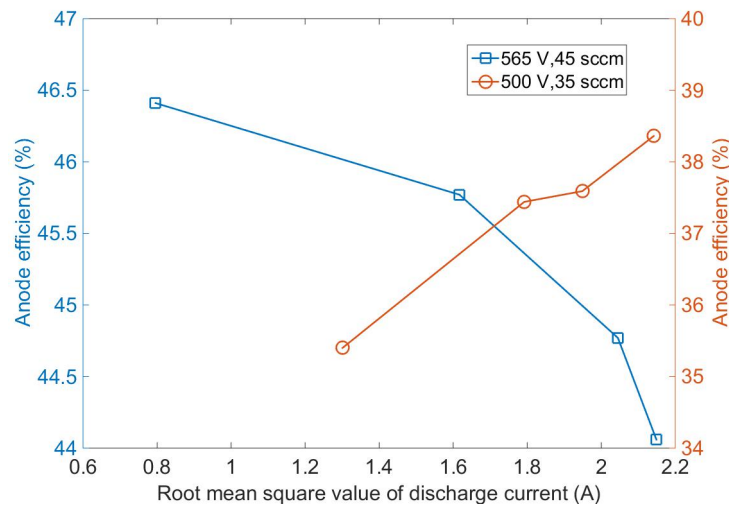


**Figure 3. Variation characteristics of the root mean square of discharge current oscillation versus capacitance**

From the effect of current oscillation on thrust and anode efficiency shown in Figs. 4 and 5, it can be seen that thrust and anode efficiency show a different change in trends with discharge current oscillation at the two operating points. Therefore, thrust and anode efficiency tend to decrease gradually in general with the gradual increase of discharge current amplitude at discharge voltage of 565 V and mass flow rate of 45 sccm; however, thrust and anode efficiencies tend to gradually increase with the gradual increase of discharge current amplitude at a discharge voltage of 500 V and mass flow rate of 35 sccm.



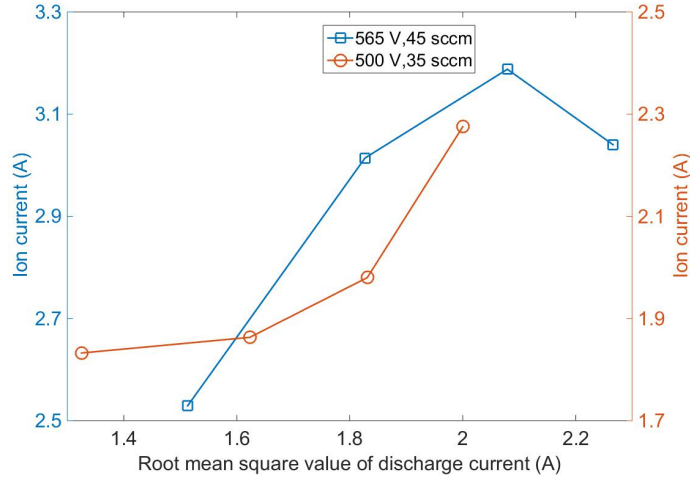
**Figure 4. Variation characteristics of thrust with low-frequency oscillation amplitude at different steady state parameters**



**Figure 5. Variation characteristics of anode efficiency with low-frequency oscillation amplitude at different steady state parameters**

The experimental results above show that the change in trend of the performance of Hall thrusters with discharge current oscillation differs at different operating points. In order to further analyze the intrinsic effect of different changes in trends of the thruster performance with current oscillation, the filter capacitance of the outer loop is changed to  $0.2\mu\text{F}$ ,  $0.3\mu\text{F}$ ,  $0.5\mu\text{F}$ , and  $1\mu\text{F}$ , at constant, steady discharge parameters. Meanwhile, the variation characteristics of ion current, propellant utilization and divergence angle of the plume during current oscillation are measured.

The discharge current during thruster operation is mainly made up of the following two parts: electron current formed by electrons reaching the anode in the channel, and ion current generated by the movement of ions formed by ionization of the gas in the channel toward the outlet. Therefore, the ion current not only represents the neutral atom ionization rate in the channel, but it is also in close relation with the variation in thruster performance. Low-frequency oscillation of the discharge current of the thruster explicitly embodies the instability of neutral gas ionization in the channel. Therefore, the analysis of ion current at different amplitudes is useful to understand the gas ionization condition in the thruster channel. During the experiment, the change of ion current with current amplitude at different operating points was measured. As shown in Fig. 6, the ion current tends to gradually increase with the increase of discharge current amplitude at both of the two operating points. However, the increase rate of ion current gradually decreases with the increase of discharge current at the discharge voltage of 565 V and the mass flow rate of 45 sccm, but gradually increases with the increase of discharge current at the discharge voltage of 500 V and the mass flow rate of 35 sccm.



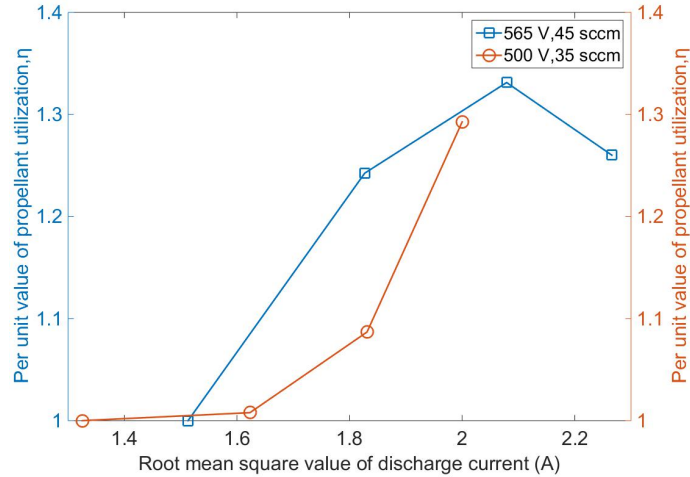
**Figure 6. Variation characteristics of ion current with low-frequency oscillation amplitude at different steady state parameters**

Propellant utilization is expressed by the ratio of mass flow rate of ions after ionization, to the anode mass flow rate of the thruster:

$$\eta = \frac{I_i M}{\dot{m} e} \quad (1)$$

Where,  $I_i$  means ion current,  $M$  denotes xenon atom mass  $M=2.2 \cdot 10^{-25} \text{ kg}$ ,  $\dot{m}$  is the mass flow rate of the anode gas, and  $e$  is the electron charge.

The variation characteristics of per unit propellant utilization with low-frequency discharge oscillation amplitude are shown in Fig. 7. From Fig. 7, it can be seen that the general change in trends of propellant utilization of the Hall thruster at different operating points are similar, i.e., the propellant utilization increases with the increase of the discharge current oscillation. To be precise, the increase rate of propellant utilization gradually increases with the increase of the current amplitude at the discharge voltage of 500 V and the mass flow rate of 35 sccm, but tends to decrease with the gradual increase of the current amplitude at the discharge voltage of 565 V and the mass flow rate of 45 sccm.



**Figure 7. Variation characteristics of per unit propellant utilization with low-frequency oscillation amplitude at different steady state parameters**

And we also measured the variation characteristic of the divergence angle of the plume with current oscillation at different operating points. To be exact, the divergence angle of plume increases from  $9.8^\circ$  to about  $13.8^\circ$  at most at the discharge voltage of 500 V and the mass flow rate of 35 sccm; the divergence angle of plume increases from  $11.7^\circ$  to about  $19.3^\circ$  at most at the discharge voltage of 565 V and the mass flow rate of 45 sccm.

#### IV. Discussions

Through the integrated analysis of the findings above, it's found that both thrust and anode efficiency of Hall thruster increase with the increase of current amplitude at the discharge voltage of 500 V and the mass flow rate of 35 sccm, thereof the thrust increases by 6.5%, and the anode efficiency by 8.4%; both thrust and anode efficiency decrease with the increase of current oscillation at the discharge voltage of 565 V and the mass flow rate of 45 sccm, thereof the thrust decreases by 1.5%, and the anode efficiency by 5.1%. With the increase of current amplitude at the discharge voltage of 500 V and the mass flow rate of 35 sccm, the ion current increases by 24.2%, and the divergence angle of plume increases by 40.8%; at the discharge voltage of 565 V and the mass flow rate of 45 sccm, the ion current increases by 20.2%, and the divergence angle of plume increases by 65.0%. The experimental results above show that the ionization in the channel intensifies with the increase of discharge current amplitude at different operating points, meanwhile, both propellant utilization increase, increasing the thrust. On the other hand, the divergence angle of plume increases with the increase of low frequency oscillation amplitude, and causes the loss of plume divergence, decreasing the thrust. At the operating point of 500 V/35 sccm, the increase rate of ion current is obviously higher than that at the operating point of 565 V/45 sccm, moreover, the increase rate of divergence angle of plume is much smaller than that at the latter operating point, i.e. the yield on ionization increase is large, and the loss of plume divergence is small, so both thrust and anode efficiency tend to increase with the increase of discharge current amplitude at the operating point of 500 V/35 sccm, but decrease with the increase of discharge current amplitude at the operating point of 565 V/45 sccm.

According to the energy balance and loss system of Hall thruster, high propellant utilization, i.e. adequate propellant ionization, must be guaranteed so that a thruster can have good overall performance. Continuity equation of neutral gas in a Hall thruster may be used to describe the ionization process of neutral gas, as shown in the following equation

$$\nabla \cdot (n_n V_n) = -\beta_i n_n n_e \quad (2)$$

Where,  $n_n$  means neutral atom density,  $n_e$  means electron density,  $V_n$  means neutral atom velocity, and  $\beta_i$  means ionization coefficient.

Morozov and others derives a similarity criterion  $S$  guaranteeing adequate propellant ionization from a zero-dimensional model<sup>[15]</sup>, and  $S$  can be expressed by the following equation

$$S = \frac{2eMU_d V_a^2}{\beta^2 L^2} \left(\frac{\dot{m}}{A}\right)^{-2} \quad (3)$$

Where,  $M$  means xenon atom mass;  $U_d$  means discharge voltage;  $V_a$  means initial velocity of xenon atoms entering the channel, i.e. 200 m/s;  $\beta$  means ionization velocity of xenon atoms, i.e.  $\beta = 2.2 \cdot 10^{-13} \text{ m}^3/\text{s}$ ;  $L$  means length of ionization region, counted as  $L=20 \text{ mm}$ ;  $\dot{m}$  means anode mass flow rate;  $A$  means cross section area of the acceleration channel of thruster.

According to the Bugrova-Maslennikov-Morozov criterion<sup>[16]</sup>,  $S$  meets the following equation

$$S < S_{cr} = \frac{1}{3} \quad (4)$$

Then the mass flow rate range guaranteeing adequate propellant ionization at certain discharge condition is acquired as below:

$$\dot{m} > \sqrt{\frac{6eMU_d V_a^2 A^2}{\beta^2 L^2}} \quad (5)$$

After the discharge parameters at the two operating points are substituted in Eq. (5), it can be seen that the lower limit of mass flow rate of adequate ionization in the channel is guaranteed at 37.4 sccm when the discharge voltage is 565 V, and the mass flow rate of neutral atoms actually flowing into the channel, i.e. 45 sccm, is much larger than such limit; the lower limit of mass flow rate of adequate ionization in the channel is guaranteed at 35.1 sccm when the discharge voltage is 500 V, and the mass flow rate of neutral atoms actually flowing into the channel, i.e. 35 sccm, is basically the same as such limit. Hence more neutral atoms are in the channel at the discharge voltage of 565 V, the frequency of their collision with electrons is higher, meanwhile, the ionization in the channel is stronger because of higher discharge voltage and larger electron energy; the density of neutral atoms in the discharge channel is relatively lower at the discharge voltage of 500 V, the frequency of their collision with electrons is lower,

meanwhile, the ionization in the channel is relatively weaker because of lower discharge voltage and smaller electron energy.

In combination with the variation characteristics of propellant utilization with current oscillation shown in Figs. 6 and 7, it can be seen that both the ion current and propellant utilization increase with the increase of discharge current amplitude at the two operating points, as indicates that the increase of current amplitude may intensify the ionization of neutral gas in the channel to some extent. At the discharge voltage of 565 V, the ion current increases with the increase of current oscillation, and the thrust decreases. Hereto, we think both discharge voltage and mass flow rate in the channel are relatively higher at this operating point, meanwhile, the initial ionization rate in the channel is relatively higher, ions generated by the ionization in the thruster are relatively more, and the thrust is larger. With the gradual increase of discharge current oscillation, although the neutral gas ionization is intensified to some extent, the increase rate of ion current decreases, moreover, the increase rate of divergence angle of plume is higher at this point, the ion energy loss is more, so both thrust and anode efficiency decrease with the increase of current oscillation. As shown in Figure 6, the increase rate of ion current decreases with the increase of discharge current amplitude, and this also indicates that the contribution rate of increase of discharge oscillation amplitude to ionization significantly decreases at a relatively higher ionization rate. At the discharge voltage of 500 V, both discharge voltage and mass flow rate are lower, initial ionization in the channel is relatively weaker, and the increase of discharge current oscillation promotes the intensification of neutral gas ionization, as a result, more ions are generated in the channel, and the increase rate of divergence angle of plume with the increase of current amplitude is smaller, at this point, both thrust and anode efficiency of thruster increase with the increase of discharge current.

## V. Conclusion

So far, many researchers have studied the oscillation characteristics through changing the steady state operating parameters of thruster, and seen the significant variation of performance, but it's still unknown whether such performance variation is caused by change of operating parameters or that of current oscillation. Hereto, this paper establishes the single association between current oscillation and thruster performance through only changing the capacitance of filter unit of Hall thruster. It studies the influence of current amplitude on thrust and anode efficiency of thruster at different operating points. The experimental result shows that the increase of current amplitude can intensify the neutral gas ionization in the channel at different operating points, but the influencing mechanism for thruster performance differs. At low voltage and small flow, the increase of discharge current promotes the intensification of ionization in the channel, ions increase, moreover, because the increase rate of divergence angle of plume is smaller, the thrust increases, so does the anode efficiency; at high voltage and large flow, although the increase of discharge current amplitude increases the number of ions a little, the increase rate of divergence angle of plume is larger, so the thrust decreases, and the anode efficiency lowers. The future work will promote the experimental and theoretical analysis of influence of higher frequency current oscillation on the performance of Hall thrusters.

## Acknowledgments

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