A Model for Turbulence-Induced Electron Transport in Hall Thrusters

IEPC-2017-367

Presented at the 35th International Electric Propulsion Conference
Georgia Institute of Technology • Atlanta, Georgia • USA
October 8 – 12, 2017

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Abstract: Many attempts have been made in the past years to address the long-standing issue of higher-than-expected electron cross-field current in Hall thrusters. The so-called "anomalous transport" phenomenon has, on one hand, considerable impacts on the overall performance of the Hall accelerators. On the other hand, the growing interest in very high-power thrusters (with discharge powers of 20kW and above) highlights once again the necessity to develop self-consistent predictive models so as to enable computer-aided design of Hall thrusters. These main motivations have given a new boost in the recent years to researches on electron dynamics and cross-field transport, in particular. One of the main findings of these investigations is providing evidence, numerical and experimental, on the major contribution that plasma turbulence has to this phenomenon. In any case, noting the non-quiescent nature of plasmas in Hall thrusters, just as in any other cross-field plasma configuration, the exact turbulent mechanisms involved and their corresponding extents remained open to further study. Nevertheless, the present effort aims at demonstrating the capability of a new approach in dealing with the problem of anomalous transport and represents a first step towards a self-consistent description of Hall thruster behavior. In the present paper, the most dominant turbulent mechanisms in the thruster inner-channel and near-plume domain were identified and their influence on cross-field current induction was captured through simplified models. These models were then unified and implemented inside a baseline quasi-2D fully-fluid simulation model of the SITAEL's HT5k thruster. This "Complemented Code" was able to provide improved predictions of plasma parameters and valuable insights into plasma turbulence in Hall thruster.

Nomenclature

A = shear factor
α_B = Bohm parameter
B = magnetic field
C_A = ion acoustic speed
E = electric field
E_θ = azimuthal electric field
E = electron charge

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γ_k = growth rate of mode k
J_{ex} = axial electron current
j_y = azimuthal electron current
K = wave number
$\boldsymbol{K}$ = wave vector
L = channel length
L_r = electron Larmor radius
M = spoke mode number
m_e = electron mass
m_i = ion mass
N = electron number density
n_0 = equilibrium electron number density
n' = perturbed electron number density
R = position vector
R_c = mid-radius of thruster channel
T = Time
T_e = electron temperature
T_i = ion temperature
u_{drift} = electron ExB drift velocity
$\boldsymbol{V}_{di}$ = ion axial drift velocity
$\boldsymbol{V}$ = electron velocity vector
v_{ex} = axial electron velocity
v_{ey} = azimuthal electron velocity
v_{AN} = anomalous collision frequency
W = wave energy
ω = instability frequency
ω_{ce} = electron cyclotron frequency
ω_{ce}^{*} = effective cyclotron frequency
ω_e = electron plasma frequency
X = axial dimension along the channel axis
θ = azimuthal dimension
φ' = amplitude of perturbed electric potential
φ_p = perturbed electric potential

I. Introduction

The interest towards Hall thrusters has raised in the past years as a result of their unique capability for application in both near-Earth and deep-space missions. Understanding the physics behind the operation of these devices can help improving their performance to make them even a more promising choice for a wide variety of space missions. However, despite several experimental, theoretical and numerical investigations to this end, there are still some open areas. One of the most important aspects that is not yet fully understood is related to the mechanisms by which electrons can diffuse across the magnetic field.

Hall thrusters provide thrust by electrostatic acceleration of ions through the potential difference imposed between anode and cathode and sustained by the presence of the applied magnetic field. The presence of the magnetic field almost perpendicular to the electric field yields an azimuthal ExB drift of the electrons around the channel. As the electrons enter their azimuthal motion, they also gain energy, collide with neutrals and ionize them, leaving behind newly-born ions and electrons. The ions, however, due to their much larger mass, remain almost unmagnetized and feel the electric field that accelerates them axially. In fact, the operation of Hall thrusters relies on the mostly-radial magnetic field near the exit of the channel. The "resistance" that electrons feel against their axial motion towards the anode, which is a result of the interaction with the magnetic field, allows the establishment of a large potential difference between the cathode and the anode. The associated electric field can accelerate ions to high velocities, up to ~30.000 m/s. Hence, the ability of the magnetic field to hinder the electrons from moving towards the anode is a factor that has a great influence on the performance of the thruster. Based on the classical transport theory, collisions between electrons and neutrals is the only mechanism enabling electrons to move forward axially. However, the axial current carried through collisions is much smaller than that measured experimentally which implies that magnetic field is less capable than expected in hindering the electron motion. Therefore, there must be other mechanisms not considered in the classical theory responsible for the enhanced electron transport across the magnetic field. The higher than expected cross-field transport is often referred to as "anomalous" transport.

Various mechanisms have been proposed as the source of this phenomenon, such as Near-Wall Conductivity (NWC) and turbulence, among which turbulence provided more consistent results with the experiments. In fact, the development of instabilities in the intrinsically non-quiescent plasma generated by Hall thrusters can provide means to electron diffusion across magnetic field lines.

The importance of providing an explanation for the anomalous transport lies in the possibility to achieve a fully self-consistent simulation model and, consequently, an aiding tool for Hall thruster design. The modeling of anomalous transport, which is the objective of the present work, serves as a preliminary step towards the development of a self-consistent predictive model that, although requiring further maturation and improvements, has yielded promising results.

Moreover, the higher-than-expected transport in Hall thrusters affects performance parameters, plasma properties and their variation along the thruster axis. For what concerns the thruster performance, the anomalous cross-field electron diffusion can significantly reduce the "current efficiency", which is one of the main source limiting the overall thrust efficiency of Hall devices. This occurs since the excess of axial electron current must be compensated by higher currents from the cathode. In other words, the higher electron transport translates into reduced resistivity faced by plasma current, decreased potential drop and ultimately the need for higher sustaining currents, which all combined impose an upper limit of about 70% on the thrust efficiency of Hall thrusters.

Considering the impact of turbulence on overall thruster performance, it is necessary to characterize the most influential mechanisms in the first place, so as to try to identify possible mitigation techniques in a second place.

II. Description of Aim and Model Review

The aim of this work is to develop a simulation code able to resolve the anomalous cross-field electron transport in a sufficiently accurate but computationally cost-effective manner. The code is to be then integrated into the baseline fully-fluid quasi-2D simulation model[31] developed at SITAEL. The combined codes will then be used to obtain more accurate predictions of plasma parameters, without the need to introduce a non-physical "Bohm parameter" inside the model.

The primary purpose of the performed modeling is to provide simplified equations capable of capturing the relevant physical characteristics of the instabilities. This approach obviates the necessity of dealing with complex kinetic behavior of the fluctuations, which can only be captured through full-PIC (Particle-in-Cell) simulations.

The modeling of the instabilities was performed in 1D, along the centerline of the thruster channel, considering the fact that variations along the axis are dominant enough to neglect possible variations of fluctuations-induced diffusivity in the radial direction.

Various instabilities, in a wide spectrum of frequencies ranging from a few kHz to MHz, have been detected in Hall thrusters, either through numerical simulations or through experiments. Each of these instabilities may only exist in a certain operating condition and/or in a particular region of the thruster. Some authors have attributed the anomalous transport to a single instability for the entire thruster domain[19,29]. However, the general idea behind the present work is, in the first place, to identify the dominant mechanism, in terms of contribution to transport, for each region of the thruster and then to model accordingly the associated electron transport in each region.

The instabilities to be modeled have been selected through an extensive literature review based on their ubiquity in different operating conditions and in various thrusters, together with well-established grounds on their role in electron transport. Table 1 shows the extent of each region and the corresponding transport mechanism speculated as dominant in the HT5k, SITAEL's 5 kW-class Hall thruster.

Region	**I** **Near-Anode**	**II** **Ionization**	**III** **Acceleration**	**IV** **Near-Plume**
Axial Location	0 – 0.42L	0.25L – 0.67L	0.67L – L	L – 2L
Transport Mechanism	**Classical Transport**	Rotating Spoke Instability	Beam-Plasma Instability	Electron Cyclotron Drift Instability

Table 1. Description of the various regions of the thruster channel with the corresponding transport mechanism; L is the channel length of the HT5k.

In modeling the anomalous transport, a common approach is to interpret the effect of current-carrying mechanisms as a collision frequency in an analogy with classical transport. Therefore, the outcome of the model will be the profile of anomalous collision frequency (υ_{AN}), which is implemented into the baseline code through an iterative process.

III. Review of the Characteristics of Selected Instabilities

This section is dedicated to a brief review of the main characteristics of the instabilities under consideration and it describes how these instabilities contribute to transport.

A. Rotating Spoke Instability

The rotating spoke is an electrostatic macroscopic instability of relatively low frequency, mostly in the range of 5-25 kHz. It appears as azimuthal fluctuations in electron number density and plasma potential propagating mainly in the $E \times B$ direction, with a phase velocity one order of magnitude lower than the electron $E \times B$ drift (about $0.2\, E_x/B_r$). The propagation of the instability, however, is not purely azimuthal and has a tilt angle of about 15-25 degrees with respect to the azimuthal plane (i.e., with a non-zero axial component of the phase velocity). The density variation was observed to be phase-correlated with the fluctuating electric field and, hence, this indicates the possibility of spoke's role in enhanced electron axial mobility[1]. Moreover, the amplitude of the fluctuating electric field (on the order of the thruster axial electric field) is reported to be sufficiently large to account for the anomalous transport in the ionization region[2].

More detailed investigation of spokes showed their weakening or absence in ionization region at higher power levels. Esipchuk et al. reported the occurrence of spoke in Hall thrusters in the low voltage part of the I–V curve and its disappearance in the current saturation part of the curve[3]. In addition, a correlation was found between the azimuthal propagation speed of the spoke and the critical ionization velocity of the propellant[7]. These observations suggested that the mechanism behind the spoke formation is the coupling between density non-uniformities and the ionization process[4]. In other words, an incomplete ionization at low voltages may cause an inhomogeneous number density distribution of charged particle in the azimuthal direction. At higher discharge voltages, the electrons gain sufficient energy to ionize the neutrals more uniformly, leading to azimuthal oscillations of smaller amplitudes[4].

Although consensus is not yet achieved on the exact explanation of the origin and sustaining mechanism of the spokes, compelling grounds exist to speculate the nature of this instability. When a minor number density of electrons coming from the cathode collide with the neutral propellant atoms on the path of their azimuthal $E \times B$ drift, in case of enough kinetic energy, the collision leads to the ionization of the atoms, producing new electrons and ions.

The newborn electrons in the ionization zone start undergoing the $E \times B$ drift. In the meanwhile, newborn ions, due to their larger inertia, can be assumed to stay stationary for longer times, readily longer than the characteristic time scale of the phenomenon described here. Nevertheless, as long as neutral atoms are present along the azimuth, newborn electrons would acquire just enough energy for their azimuthal speed to reach, at maximum, that corresponding to the Alfven critical ionization velocity (CIV). Further acceleration from the electromagnetic force serves as providing the energy necessary for the ionization of the neutral atoms by newborn electrons. As a result, these azimuthally propagating electrons form an azimuthal ionization front, which is a characteristic of the rotating spokes. On the other hand, however, as these electrons start leaving the first ionization zone, charge separation generates an azimuthal electric field (E_θ), which increases up to a certain point and then decreases as the electrons start feeling the presence of the ions in another ionization zone. This results in the oscillation of the wave electric field of the instability. Based on the large number of spoke characterization efforts, we know that the velocity that the electrons acquire is lower than the CIV, probably because the azimuthal electric field caused by the charge separation also slows down the electrons and accelerates the ions. In accordance with this possible explanation, Sekerak proposed, from the energy balance, to consider the azimuthal rotating spokes as an ambipolar diffusion[5]. Ambipolar diffusion sets an electric field that accelerates ions to the Bohm velocity and decelerates the electrons[5] and, hence, the ions gain energy across this moving "sheath"[6].

To explain the mechanism by which spokes carry axial current, it can be noted that the azimuthal fluctuating electric field E_θ together with the applied radial magnetic field B_r drives an axial $E_\theta \times B_r$ electron drift that changes its direction around the circumference of the channel. Nevertheless, as the perturbations in density and electric field associated with the spoke are phase-correlated, integrating the induced current over the channel circumference gives a non-zero axial current, implying an enhancement in cross-field electron transport towards the anode.

Several efforts evidenced the spoke's role in anomalous transport. The experiments conducted by McDonald using a 12-element segmented anode on the H6 Hall thruster concluded that approximately 50% of the electron current in the near-anode region is carried by rotating spokes[7]. This result is consistent with the works of Ellison and Lomas[8,9].

B. Beam-Plasma Instability

Beam-plasma is a high frequency (about 2-5 MHz) axially-propagating longitudinal instability with wavelengths on the order of the electron Larmor radius. This instability belongs to the general category of two-stream instabilities and thus, one of its excitation criteria is the existence of a sufficiently large relative drift between electrons and ion beam, which is met in Hall thrusters. In fact, the reduced axial mobility (and hence, lower velocity) of electrons in Hall devices together with the presence of a high-speed ion beam can result in a two-hump distribution function of the plasma, which gives rise to the Beam-plasma instability[10]. The excitation of Beam-plasma instability can be also due to Secondary Electron Emission from the channel walls. In this case, the SEE is required to reach a level corresponding to the space-charge saturated sheath[18].

According to Thomas and Cappelli, a possible mechanism for the development of this instability can be inferred from the fact that the phase velocity of the unstable root in the dispersion relation is comparable to that of the fast transit-time instability near the exit[10]. Accordingly, they suggested that the Beam-Plasma instability can be considered as a high-frequency manifestation of the general transit-time instability[11,12].

Beam-Plasma instability contribution to transport is explained based on the "electron cyclotron orbit distortion" theory[10]. The distortion of electrons' orbit occurs as a result of the $E \times B$ shear due to either spatial variation in static electric field over a Larmor radius or the presence of strong electrostatic fluctuations. In the case of Hall thrusters, the distorted cyclotron orbits become elliptical in such a way that the major axis is oriented along the channel axis. In this situation, larger electron Larmor radius along the axial electric field corresponds to less magnetization of electrons in this direction, resulting in a larger cross-field electron current towards the anode. This is an effect that is commonly referred to as the "neoclassical" transport.

The effect of the orbital distortion can be described through the introduction of a shear factor α and an effective cyclotron frequency ω_{ce}^* as follows[10]

$$\alpha = \hat{x} \cdot \nabla \times \left(\frac{E \times B}{B^2}\right) \frac{1}{\omega_{ce}} \sim \frac{1}{\omega_{ce}\, B} \left(\frac{dE}{dx}\right), \tag{1}$$

$${\omega_{ce}^*}^2 = {\omega_{ce}}^2 (1 - \alpha). \tag{2}$$

Based on above equation, the sign of α determines its impact on transport:

a) If $\alpha < 0$, cyclotron orbits are squeezed in the axial direction leading to a reduced mobility.
b) if $0 < \alpha < 1$, cyclotron orbits are stretched in axial direction resulting in enhanced transport, which is the case expected to happen in Hall thrusters.

C. Electron Cyclotron Drift Instability

Electron Cyclotron Drift Instability (ECDI) owes its name to the fact that the instability waves have wavelengths on the order of the electron Larmor radius and that they propagate mainly in the $E \times B$ direction, i.e. in the direction of the electron drift.

ECDI is a high frequency instability falling in a frequency range of 1-10 MHz and is due to the coupling between electron Bernstein (electrostatic) modes and ion acoustic waves[13]. The phase velocity is smaller than or at most equal to the $E \times B$ drift speed of electrons and has been seen to occur, in most cases, just outside of the exit plane of the thruster, in the region of negative gradient of magnetic field[32,33].

It is known that this instability grows very fast and saturates rather quickly[26], which is the detectable state in simulations and experiments. Since the saturation state is characterized by high frequency and large amplitude waves, this instability is capable of causing large cross-field transport and significant heating of the plasma. Moreover, it strongly modifies the Secondary Electron Emission (SEE) regime from the channel walls and plays an important role in exciting other plasma wave modes, specifically axially-propagating longitudinal ones[18].

This instability is the dominant current-induction mechanism in the near-field plume region of a wide variety of Hall thrusters[34,35], yielding anomalous collision frequencies at least one order (up to three orders) of magnitude higher than that inside the thruster channel[24,26,29]. In fact, due to the remarkable involvement of ECDI in cross-field transport enhancement, its inclusion in modeling and integration into the simulation code plays a more important role, in increasing the self-consistency of the simulation, than that of the two aforementioned instabilities.

The significance of the ECDI contribution to anomalous transport roots in the multitude of consequences its presence has on the behavior of the plasma. It is speculated that there exist three main effects related to ECDI that help electrons to overcome the magnetic barrier. These effects are briefly explained in the following:

1) During the rapid nonlinear growth of the instability, the strong wave-particle interactions result in heating of the bulk plasma in all directions. This heating, which flattens to some extent the electron velocity distribution function, is tantamount to the diffusion of the electrons in velocity space. This diffusion means that the number density of electrons with larger thermal velocities increases such that the radius of the electrons' cyclotron orbits collectively increases, yielding a net drift of particles towards the inner regions of the channel. However, it must be mentioned that the detailed explanation of ECDI-induced transport through this mechanism can be more complex than what described here.
2) As ECDI generates fluctuations in both azimuthal electric field and charged particle number density, the existence of a correlation between the electric field and density oscillations, results in an enhanced electron-ion "*friction*" (collisions)[14]. In fact, the electron-ion friction is proportional to the term $\langle \delta n_e \delta E_\theta \rangle$ and the value of this average is determined by the instability characteristics. Hence, the augmented electron-ion "*friction*" can be directly responsible for the increased electron transport[14,15,16].
3) The heating of the bulk plasma by the ECDI modifies the SEE regime and, as a result, the near-wall sheath becomes space-charge saturated. As the saturated sheath is shown to be unstable[17], some axially-propagating longitudinal electrostatic waves of high frequency are excited whose electric fields enhance the cross-field transport of electrons. Some of these wave modes are numerically shown to be due to the excitation of the Beam-plasma instability near the channel walls when the sheath enters the space-charge saturated regime[18].

After the rapid nonlinear growth of the ECDI, the wave modes of the instability enter a pseudo-saturation state[25] and this transition marks the excitation of the ion acoustic instability, characterized by azimuthally-propagating waves of much lower growth rates. As this occurs, the ion acoustic waves maintain the large cross-field current induced by the ECDI modes while also adding to this current, albeit at a much slower rate corresponding to their slower growth. In the region where ion acoustic waves are present, the electrons drift velocity remains large enough to drive the growth of large amplitude ion sound waves, which are excited as a result of ECDI transition. The instability grows by gaining energy from azimuthal motion of electrons; therefore, it acts as a "drag" on the electron drift that, in turn, can be modeled as an effective collision frequency. The generated ion acoustic waves move downstream with the supersonic beam of ions to the near-plume region[19].

Noting the difficulty in capturing the effects on anomalous transport from the nonlinear regime of the ECDI, the theory that accounts for the role of the ion acoustic modes has been implemented in the model.

IV. Underlying Theories of Instabilities

In this section, the theories governing the spatial evolution of each turbulent transport mechanism developed by other authors previously are presented. In the formulation, the temporal variations are not considered so as to obtain a description of the instability waves consistent with the baseline code. Excluding the time dependence from the equations, however, does not seem to cause any considerable error in the results and would describe the overall physical picture with sufficient accuracy. In fact, omitting in-time variations can be thought of as a time-averaged description of the turbulent phenomena.

Despite the above point, the instabilities are inherently time-dependent phenomena and therefore, excluding time from the equations governing their evolution has to be compensated by externally introducing some of their physical characteristics in the theories in order to minimize any possible loss of physics.

A. Rotating Spoke Instability

The theory explaining the electron transport through azimuthal electric field and number density fluctuation was originally developed by Yoshikawa[20] and also discussed by McDonald[7].

Consider the perturbed azimuthal electric field and perturbed electron number density to be of the following form, respectively

$$E_\theta = E'_\theta \sin(\theta), \tag{3}$$

$$n = n_0 + n' \sin(\theta). \tag{4}$$

The azimuthal fluctuating electric field together with the radial magnetic field of the thruster induces an alternating axial current as

$$J_{ex}(\theta) = nev_{ex} = ne\frac{E_\theta}{B} = \frac{e}{B}[n_0 E'_\theta \sin(\theta) + n' E'_\theta \sin^2(\theta)]. \tag{5}$$

The average axial current $\bar{J}_{ex}$ is obatined through integration over the azimuthal angle

$$\bar{J}_{ex} = \frac{1}{2\pi}\int_0^{2\pi} J_{ex}(\theta)\, d\theta = \frac{e}{2B} n' E'_\theta. \tag{6}$$

Assuming n' and E'_θ to be perfectly in-phase, the relation between the two is given by the following equation

$$E'_\theta = \frac{1}{4}\pi \frac{n'}{n_0} E_x, \tag{7}$$

and, by substitution in Eq. (6), the average axial electron current carried by spoke can be obtained in terms of number density perturbation

$$\bar{J}_{ex} = \frac{e\pi}{8}\frac{E_x}{B}\frac{(n')^2}{n_0}. \tag{8}$$

As also described previously, the electric potential drop of the spoke can be interpreted as a moving sheath[6,21]. Hence, the magnitude of the drop is the same as that required for accelerating the ions to the ion acoustic speed (Bohm velocity). In this sense, Sekerak also proposed to consider the spokes azimuthal electric field as that necessary for the ambipolar diffusion[5]. These observations can be used as a condition to estimate the amplitude of the azimuthal fluctuating electric field associated with spokes. Therefore, the energy conservation implies that the ion kinetic energy, corresponding to the ion acoustic velocity, has to be equal to the potential energy through which they have been accelerated

$$e\varphi_\theta = \frac{1}{2} m_i C_A^2, \tag{9}$$

where C_A is the ion acoustic velocity

$$C_A = \sqrt{\frac{K_b T_e}{m_i}}. \tag{10}$$

Given the amplitude of the potential fluctuation φ'_θ, the amplitude of the fluctuating electric field can be written as

$$E'_\theta = k\varphi'_\theta, \tag{11}$$

where k is the wave number of the dominant mode. Various observations in common annular Hall thrusters with power levels on the order of 5-6 kW have detected the mode number of 3 ($m = 3$) as the dominant spoke mode[2,7]. The wave number 'k' corresponding to the mode number 'm' is

$$k = \frac{m}{R_c}, \tag{12}$$

in which R_c is the mid-radius of the thruster channel.

Finally, the anomalous collision frequency associated with the spoke instability is obtained from the following equation

$$\upsilon_{AN} = \omega_{ce}\left(\frac{\bar{J}_{ex}}{j_y}\right), \tag{13}$$

where j_y is the azimuthal electron drift current (Hall current) given by the following equation

$$j_y = en_0\frac{E_x(x)B_r(x)}{B^2(x)}. \tag{14}$$

B. Beam-Plasma Instability

As described before, the mechanism by which the Beam-Plasma instability enhances the cross-field electrons diffusivity is explained by the cyclotron orbit distortion theory. The application of this theory to the case of Hall thruster has been provided by Thomas and Cappelli[10,22]. The same approach was followed in this work.

The derivation procedure is started with the equation of motion for a single charged particle in a crossed E and B configuration, i.e.

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m}(\boldsymbol{E} + \mathbf{v} \times \boldsymbol{B}). \tag{15}$$

From this equation, neglecting space dependency of the electric field and considering a velocity in the plane perpendicular to the magnetic field, we obtain

$$\frac{d^2\mathbf{v}}{dt^2} + \omega_{ce}^2\mathbf{v} = \frac{e}{m}\frac{d\boldsymbol{E}}{dt} + \left(\frac{e}{m}\right)^2 \boldsymbol{E} \times \boldsymbol{B}. \tag{16}$$

The term $\frac{d\boldsymbol{E}}{dt}$ represents the polarization drift whenever electric field is considered as a varying function of time. However, in case of Hall thrusters, for which the spatial variation of electric field is significant, this term can be written as

$$\frac{d\boldsymbol{E}(\boldsymbol{x},t)}{dt} = \nabla_{\boldsymbol{x}}\boldsymbol{E}|_t \cdot \mathbf{v} + \left.\frac{d\boldsymbol{E}}{dt}\right|_{\boldsymbol{x}}. \tag{17}$$

which yields, by substitution into Eq. (16), the altered trajectory of the particles due to spatial gradient of the electric field (or equivalently, spatial gradient of the drift velocity)

$$\frac{d^2\mathbf{v}}{dt^2} + \left(\omega_{ce}^2\mathbb{I} - \frac{e}{m}\nabla_{\boldsymbol{x}}\boldsymbol{E}|_t\right) \cdot \mathbf{v} = \frac{e}{m}\left.\frac{d\boldsymbol{E}}{dt}\right|_{\boldsymbol{x}} + \left(\frac{e}{m}\right)^2 \boldsymbol{E} \times \boldsymbol{B}. \tag{18}$$

On the left-hand side, all of the terms in parentheses together serve as the modified effective cyclotron frequency (ω_{ce}^*) already introduced in Eq. (2).

The above equation shows that, whenever spatial gradients of the drift velocity (the second term in parentheses) are comparable to the cyclotron frequency, the effect would be significant in modifying the electrons' trajectories. Note that the orbit distortion affects all electrons coherently and, consequently, this microscopic effect can imply a macroscopic cross-field transport.

Under the assumption of a much smaller perturbation frequency compared to the cyclotron frequency, the solution of Eq. (18) in the x-direction (parallel to electric field gradient), which is of more importance to us, is

$$v_{ex} = \frac{\frac{e}{m}\left.\frac{dE_x}{dt}\right|_{\boldsymbol{x}}}{{\omega_{ce}^*}^2} + \frac{\left(\frac{e}{m}\right)^2(\boldsymbol{E} \times \boldsymbol{B})_x}{{\omega_{ce}^*}^2} + S\cos(\omega_{ce}^* t - \phi), \tag{19}$$

in which S is the cyclotron orbital speed. Noteworthy is the fact that the second term in the above equation requires a resolved azimuthal fluctuating field, which is not considered in our model, as we are only concerned with axial electrostatic fluctuations. Hence, the axial electron speed can be written as the summation of drift due to modified electron-neutral collision frequency and polarization drift

$$v_{ex} = V_{en} + V_{pd} = \frac{\omega_{ce}\,v_{en}}{{v_{en}}^2 + {\omega_{ce}^*}^2}\left(\frac{E}{B}\right) + \frac{\frac{e}{m}\left(\frac{dE_x}{dt}\right)}{{\omega_{ce}^*}^2},$$

where v_{en} is the electron-neutral classical collision frequency resolved by the baseline code. The effective cyclotron frequency is obtained by substituting the following equation for shear factor proposed by Thomas, which reflects the significant effect of axial fluctuations on orbit distortion[22]

$$\alpha = (kL_r)^2\left(\frac{e\varphi'}{K_bT_e}\right), \tag{20}$$

where L_r is the electron Larmour radius and φ' is the amplitude of the electric field fluctuation, given by

$$\varphi' = \frac{1}{k}E'. \tag{21}$$

The value of "k" is chosen for the mode having the maximum power intensity in the acceleration region, the accurate determination of which requires PIC simulations or experiments. However, based on the results of other studies, we considered this value to be ~450 m^{-1}. This value corresponds to a wavelength for the instability waves on the order of the Larmour radius, which is one of the characteristics of the Beam-Plasma instability[22,23]. The frequency of the instability is also obtained by solving the dispersion relation[22].

Another point is that the instability grows until the fluctuation amplitude of the wave becomes large enough to trap part of the beam ions in its potential troughs. This situation, as stated previously, marks the saturation of the instability wave. Since saturation is a time-dependent process and we have omitted time-varying phenomena in our modeling, the effect of the fluctuation amplitude growth is modelled through the introduction of a constant equivalent amplitude. As reported in a number of publications, specifically in those by Thomas et. al.[23,23], this amplitude has been observed to be as large as 2-2.5 times the average (over the whole channel length) of the static electric field. We assume E' to be two times the average of the static electric field in the thruster.

Finally, the increased axial drift of electrons can be related to an anomalous collision frequency using the following equation

$$v_{AN} = \omega_{ce}\left(\frac{v_{ex}}{v_{ey}}\right), \tag{22}$$

in which v_{ey} is the azimuthal drift of electrons corresponding to the Hall current.

C. Electron Cyclotron -drift Instability (ECDI)

In this work, we used the theory developed by Katz et al.[19,24], which attributed the ECDI contribution to transport to the second mechanism described in the previous section. The energy that ECDI takes from the electron flow in azimuthal direction can be interpreted as drag and, consequently, it can be related to a collision frequency pushing electrons forward axially. The value of this opposing force depends on the term $\langle \delta n_e \delta E_\theta \rangle$, which has a non-zero value due to the presence of an azimuthal instability inducing correlated fluctuations in the electric field and electron number density[14,15].

In the model, it is required to first obtain a relation between the azimuthal "*drag force*" on the electrons and the perturbations in plasma parameters and, then, to determine the axial variations of this force or, alternatively, the corresponding collision frequency.

Under the assumptions of large Hall parameter, $u_{drift} \gg C_A$, $T_e \gg T_i$ and also considering that the ion acoustic instability's wave vector is mainly azimuthal, the anomalous collision frequency can be described in terms of the wave energy W as

$$v_{AN} = \left[\frac{e}{n_0}\right]\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{1}{m_i m_e}\right)^{1/2} k^2 W. \tag{23}$$

To obtain the evolution of anomalous collision frequency along the channel, we can use the wave kinetic equation

$$\frac{\partial W_k}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left[\frac{\partial \omega}{\partial \boldsymbol{k}}W_k\right]-\frac{\partial}{\partial \boldsymbol{k}}\cdot\left[\frac{\partial \omega}{\partial \boldsymbol{r}}W_k\right]=2\gamma_k W_k. \tag{24}$$

Considering the dispersion relation of ion acoustic wave in the laboratory frame to be

$$\omega = C_A k+\boldsymbol{k}\cdot\boldsymbol{V}_{di}, \tag{25}$$

and assuming that in the near-plume region $V_{di} \gg C_A$, Eq. (24) can be simplified to the following form

$$\frac{\partial W}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\cdot[\boldsymbol{V}_{di}W]=2\gamma_k W. \tag{26}$$

Finally, substituting W in above equation using the expression obtained for ν_{AN}, the evolution equation for ν_{AN} in time and space is found to be

$$\frac{\partial[n_0\nu_{AN}]}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\cdot[\boldsymbol{V}_{di}n_0\nu_{AN}]=2\gamma_k n_0\nu_{AN}. \tag{27}$$

In the above equation γ_k is the growth rate of ion acoustic instability that, under the assumptions of $u_{drift} \gg C_A$, $T_e \gg T_i$, can be approximated as

$$\gamma_k = k\sqrt{\frac{\pi}{8}}\sqrt{\frac{m_e}{m_i}}u_{drift}, \tag{28}$$

which satisfies the following criterion

$$k_\theta u_{drift} \approx n\omega_{ce}. \tag{29}$$

Based on the simulations conducted by Ducrocq et al.[25] and Adam et al.[26], the **fastest growing mode** has the most substantial effect on the deformation of the distribution function and, hence, on the electron transport. Therefore, this mode, which is the n-th harmonic of the cyclotron frequency given by

$$n_{fast}=\frac{1}{\sqrt{2}}\left(\frac{u_{drift}}{u_e}\right)\left(\frac{\omega_e}{\omega_{ce}}\right), \tag{30}$$

is incorporated in the model. As suggested in Ref. 27, we assumed that the saturation of instability occurs when the anomalous collision frequency reaches the electron cyclotron frequency.

The collision frequency profile obtained from the first iteration with the baseline code is presented for an operating condition corresponding to 300V of discharge voltage and 16.5 mg/s of mass flow rate. The different regions, their extent and the dominant current-carrying instability in each region correspond to those reported in Table 1. As it can be seen in Figure 1, in line with Ref. 36, a characteristic dip appears near the channel exit, corresponding to the location of maximum magnetic field intensity; here the value of anomalous collision frequency approaches that of electron-neutral collision. Together with the rapid rise of about three orders of magnitude in the near-plume region, this behavior can serve as the primary validation for the UATC result, since the presence of these features in the anomalous collision frequency profile can lead to improvements in the computed plasma properties.

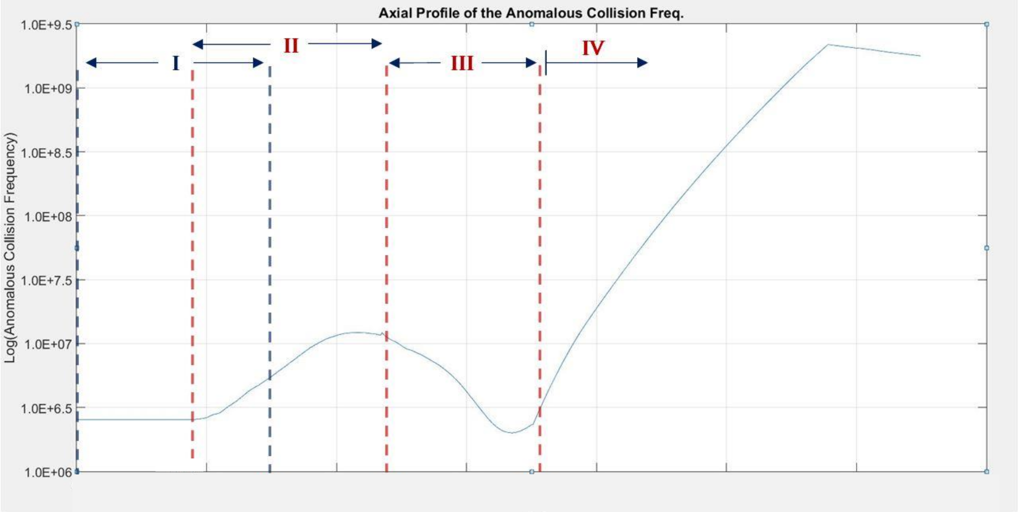


Figure 1. The axial profile of the anomalous collision frequency obtained from the unified anomalous transport code

V. Discussion of UATC Implementation Result

The UATC is integrated into the baseline code in order to provide a more self-consistent and more accurate prediction of plasma properties. The iteration process is initiated assuming a constant Bohm parameter for the entire simulation domain and it proceeds by taking the average of the collision frequency profiles of the two previous steps as the guess for the subsequent step. The process continues until it converges to a solution.

One important aspect to note is that the division of the inner channel into ionization and acceleration regions is based on the axial profile of plasma number density. More specifically, the axial location at which the peak of number density profile occurs marks the boundary between the ionization and acceleration regions. Accordingly, after each iteration, the location of transition from the spoke to the Beam-Plasma instability was updated with respect to changes in the plasma density profile.

Figure 2 shows the final anomalous and effective collision frequency profiles obtained after the convergence of the Complemented Code. These profiles and all subsequently presented results are obtained for a discharge voltage of 300V and a mass flow rate of 16.5 mg/s. Note that, in all of following figures, the red dashed-line identifies the location of the channel exit.

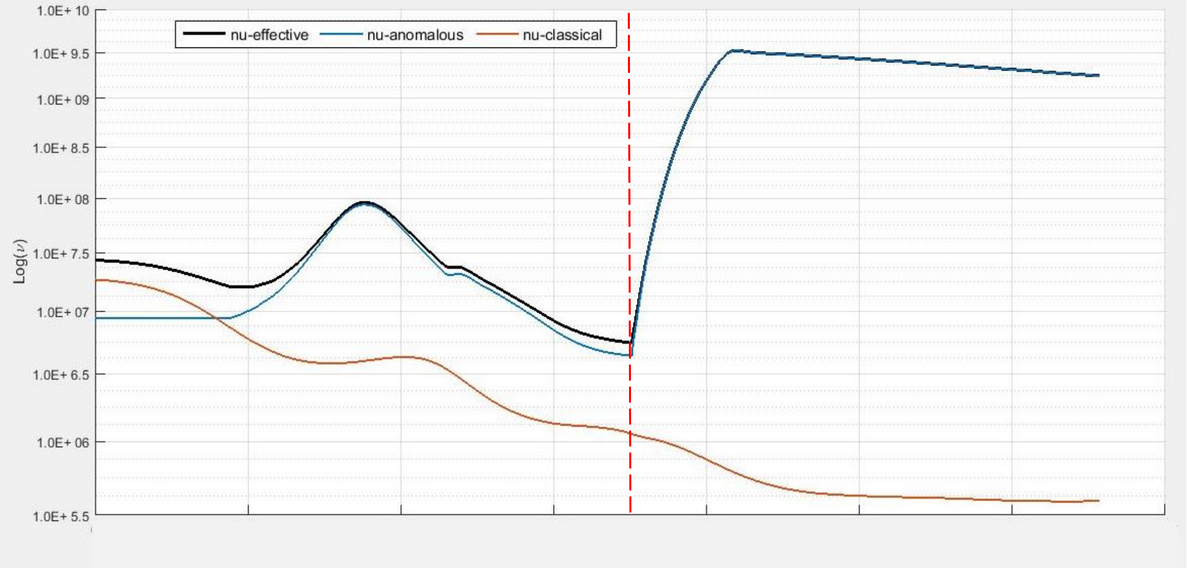
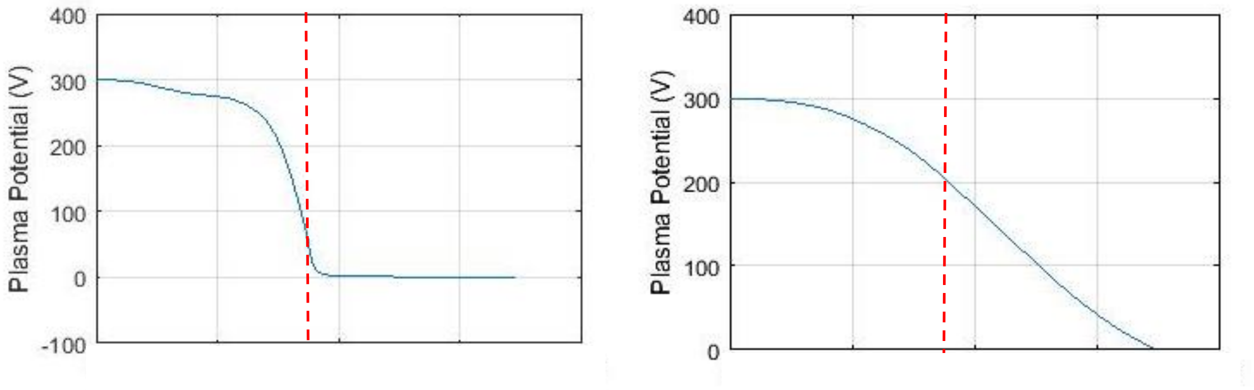


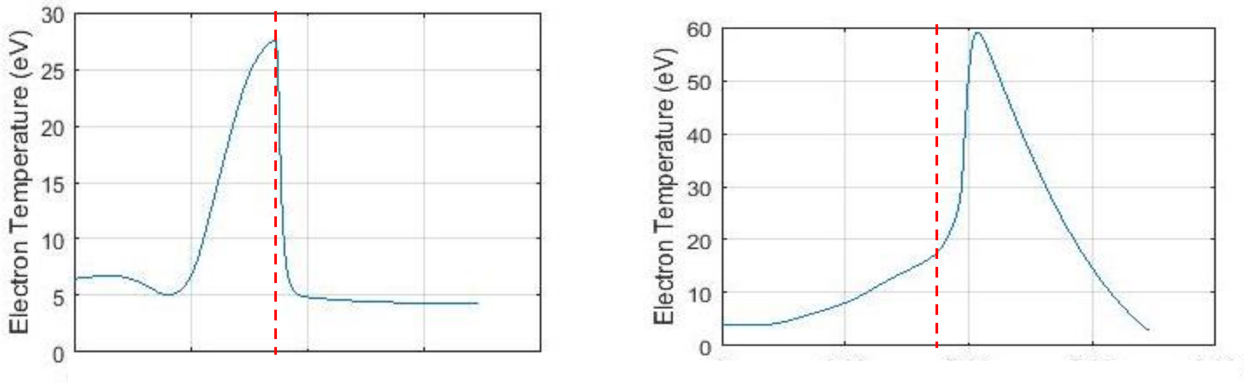
Figure 2. Final profiles of effective, anomalous and electron-neutral collision frequencies from the "Complemented Code"

In the following, we compare the 1D profiles of electron temperature and plasma potential from the Complemented Code with those from the baseline (See Figure 3). It is reminded that in the baseline code the effect of anomalous transport has been approximated by introducing a Bohm collision frequency such that $\nu_{AN} = \alpha_B \omega_{ce}$, with a constant Bohm parameter α_B for the entire simulation domain.



(a) (b)

Figure 3. Comparison between the plasma potential profile from (a) Complemented and (b) Baseline codes



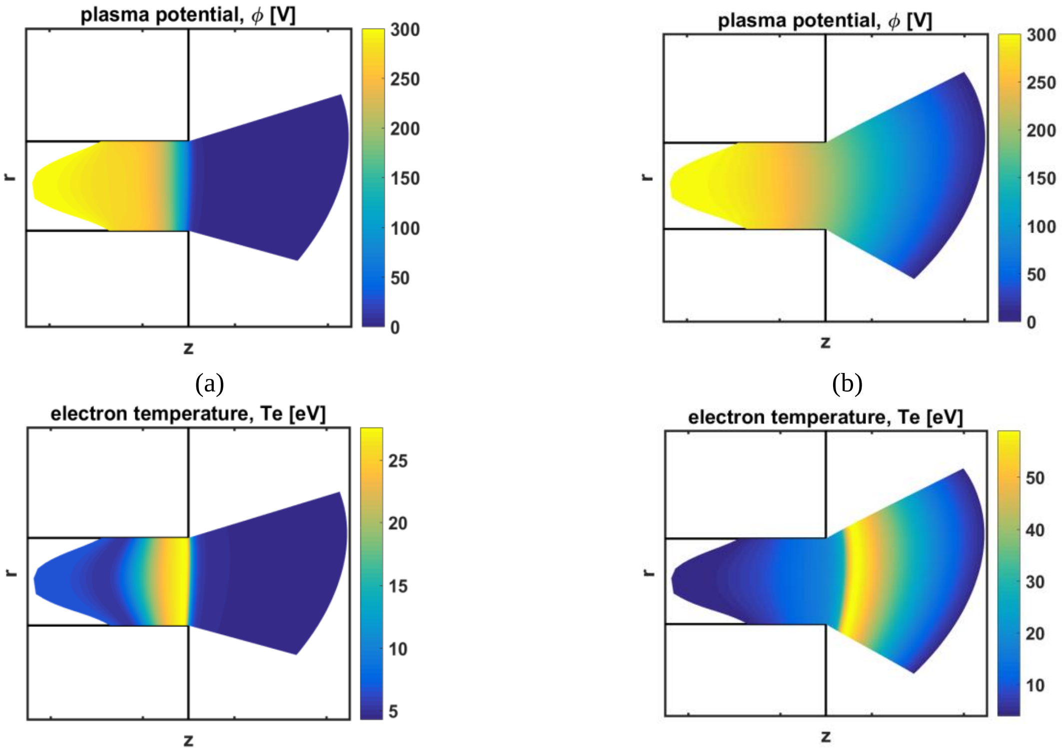
(a) (b)

Figure 4. Comparison between the electron temperature profile from (a) Complemented and (b) Baseline codes

As it can be seen, the presence of a dip in the effective collision frequency profile implies a rapid change in the plasma potential, since the reduced transport in the vicinity of the dip must be compensated by an increased potential drop, which aids the electrons in overcoming the strong magnetic field barrier. Clearly, this physical feature of Hall thruster operation captured by the Complemented Code is absent in the profile predicted by the baseline code. It is immediately recognizable in Figure 3(b) that low values of the collision frequency, like those considered for the plume region in the baseline code, result in an essentially unrealistic prediction of the electric potential. This once again highlights that the Bohm diffusion model is not sufficient to capture the anomalous conductivity in the near-plume region and that the electron mobility in the near-plume region, at least over a certain length after the exit, cannot be scaled with a $1/B$ dependency[14,15,16].

It is also noteworthy to point out the consistency between the profiles of electron temperature and plasma potential as shown in Figure 3(a) and Figure 4(a). Indeed, the rapid decrease in the potential from the beginning up to the end of the acceleration region results in a decrease in the electron temperature.

In addition, the Complemented code provides a more realistic prediction of the electron temperature: looking at Figure 4(a), the peak of electron temperature is lower and shifted inside the channel. This is consistent with the usual characteristics of Hall thrusters with conventional magnetic field configurations, in which the peak of the temperature occurs inside the channel and is limited to values in the range of 30 to 40 eV, due to the presence of the ceramic walls[29,30]. These improvements can be better visualized through the 2D profiles shown in Figure 5.



(a) (b)

(c) (d)

Figure 5. 2D Profiles of plasma potential and electron temperature from Complemented code, (a) and (c), and the Baseline code, (b) and (d).

An interesting point revealed by the iteration results is that, apart from the impact of turbulence in the exit and near-plume region, the values of anomalous collision frequency inside the channel also have a significant influence on the behavior of intensive plasma parameters, as well as on the overall performance parameters such as the discharge voltage (V_D). In fact, given the discharge current, the difference between the value of ν_{AN} corresponding to the beginning of acceleration region and the dip in Figure 2 appreciably affects the potential drop and consequently the electric field.

In spite of the sensitivity of the results on the accuracy of the turbulence models **inside** the channel, the satisfactory improvements that the implementation of the instabilities models provide in the prediction of plasma properties, even under the mentioned simplifying assumptions, reveals an interesting insight regarding the spoke instability. The results of the Complemented code (see Figure 2 and 3(a)) are compatible with the interpretation of the spoke instability as a moving "presheath"[5] that has an electric field amplitude on the order of that necessary to accelerate ions to the Bohm velocity.

Another important point to note is that the Beam-Plasma Instability, with the assumed electric field amplitude, as a mere transport mechanism in the acceleration region is observed to be insufficient. The assumed parameters in this region resulted in a much lower dip in the effective collision frequency profile and, therefore, a larger potential drop. This might be due to the lack of an accurate modeling of the sheath and the corresponding SEE[18]. As a matter of fact, a complex coupling exits between the physics of the bulk plasma and that governing the Secondary Electron Emission from the walls. Hence, an appropriate model of near-wall sheath is needed to take into account the excitation of the Beam-Plasma instability as a result of the change in the SEE regime from the walls to that of the space-charge-saturated and the presence of the ECDI, which heats the bulk plasma in all directions.

Another explanation may be that the electron-wall collisions and near-wall conductivity play a non-negligible role in the acceleration region and accommodating these effects into the code might account for the additional required anomalous collision frequency. This explanation is, in part, in line with the results of fully-kinetic simulations performed by Coche and Garrigues, in which the role of electron-wall collisions in transport has been shown to be considerable at least within a certain timeframe during the current rise in the oscillatory pattern of the discharge current[28]. In our code, either increasing the strength of the Beam-Plasma or augmenting the contribution of electron-wall collisions have shown to address this issue equivalently right.

VI. Conclusion

In this paper, a new approach to address the problem of anomalous electron cross-field transport was reported. One of the objectives of this effort was to demonstrate that, based on the existing physical description of relatively well-characterized plasma instabilities in Hall thrusters and even under simplifying assumptions, it is possible to acquire more accurate, self-consistent predictions of plasma properties. The developed model, UATC, not only captured a number of physical features pertained to the Hall thruster operation but also its integration into the baseline model provided valuable information regarding the turbulent nature of Hall thruster plasma and the role of turbulence in electron cross-field transport.

Nevertheless, room to further improve the UATC and Complemented code still exists. It was mentioned in the previous sections that the theories used to model the contribution of chosen instabilities to transport were supported by an in-advance knowledge of some relevant physical characteristics of the instabilities obtained from other experimental and/or numerical analyses. UATC can become more mature if the descriptions are modified such that at least part of these insights into the nature of the instabilities is resolved within the code itself.

Furthermore, an important step towards a higher degree of accuracy and self-consistency is to allow the codes to resolve the time variations. If non-stationary conditions are investigated, certain characteristics of the instabilities waves, such as growth rate and saturation, can be better described. This is probably of more importance in case of Beam-Plasma instability, as its behavior is more sensible with respect to temporal variations[23]. In any case, as presented above, the models used to relate the anomalous cross-field current to physical properties of Spoke instability and ECDI allowed a significant improvement of the code predictions even in their steady-state description. This implies that even simplified models like those used in UATC for instabilities are sufficient to account for the anomalous electron transport.

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