Laser-Induced Fluorescence Measurements of the Acceleration Zone in the 12.5 kW HERMeS Hall Thruster

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Non-invasive measurements of the ion velocity distribution function (IVDF) as a function of position using laser-induced fluorescence (LIF) are a critical component of life qualification of the 12.5 kW Hall Effect Rocket with Magnetic Shielding (HERMeS), providing the empirical inputs necessary to determine the anomalous cross-field transport profile for hydrodynamic simulations of the thruster using the Hall2De code. This paper presents LIF measurements of singly ionized xenon velocities along the discharge channel centerline and across a grid of points spanning the outer half of the channel and near plume, for HERMeS operation at 20.83 A discharge current and 300, 400, 500, and 600 V discharge voltages. The plasma potential profiles at the four discharge voltages approximately overlapped in the region with plasma potential less than 300 V; ion acceleration began further upstream at higher discharge voltages because the region with a steep potential gradient was broader. Bimodal IVDFs were measured in the acceleration zone at discharge voltages of 500 and 600 V; this effect was attributed to time-averaging over movement of the acceleration zone during large-amplitude discharge current oscillations at these discharge voltages. The acceleration zone was located further upstream at higher magnetic field strengths and also moved upstream slightly with increasing background pressure. Two-dimensional vector maps of mean ion velocity revealed greater radial divergence of ion trajectories at 300 V than at higher discharge voltages; this observation may help to explain the high inner pole erosion rate at this operating condition. The paper concludes with an appendix that clarifies some commonly misunderstood points about LIF saturation.

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Nomenclature

\( z \) = axial coordinate (parallel to the thruster centerline)
\( r \) = radial coordinate
\( v_z \) = axial ion velocity
\( v_r \) = radial ion velocity
\( V_d \) = discharge voltage
\( I_d \) = discharge current
\( V_p \) = plasma potential
\( B \) = normalized magnetic field strength
\( B_{\text{nom.}} \) = nominal magnetic field strength for the HERMeS thruster
\( v \) = ion velocity
\( u_i \) = mean velocity for all ions
\( c \) = speed of light
\( \lambda_0 \) = LIF transition line center wavelength in the ion rest frame
\( \lambda_{\text{laser}} \) = laser wavelength
\( f_i(v) \) = ion velocity distribution function (IVDF)
\( f_1(v) \) = Maxwellian fit to IVDF data
\( f_2(v) \) = bi-Maxwellian fit to IVDF data
\( A_1, A_2 \) = amplitude fit parameters
\( v_1, v_2 \) = mean ion velocity fit parameters
\( v_{T1}, v_{T2} \) = ion thermal velocity fit parameters
\( \langle v \rangle \) = mean velocity calculated from curve fits or averaged over an ensemble of particles
\( v_{\text{east}} \) = ion velocity along “east” injection beam direction
\( v_{\text{west}} \) = ion velocity along “west” injection beam direction
\( \phi \) = plasma potential
\( \phi_{\text{anode}} \) = anode potential
\( m_i \) = xenon ion mass
\( e \) = electron or ion charge
\( \mathbf{x} \) = position vector
\( t \) = time
\( L_{\text{channel}} \) = HERMeS thruster channel length
\( \text{Xe II or Xe}^+ \) = singly ionized xenon
\( \text{Xe III or Xe}^{2+} \) = doubly ionized xenon
\( \omega \) = oscillation angular frequency
\( v_0(t) \) = time-dependent mean ion velocity in the presence of breathing mode oscillations
\( \Delta v_0 \) = amplitude of oscillations in the mean ion velocity
\( T_i \) = ion temperature
\( k_B \) = Boltzmann’s constant
\( N \) = number of ions
\( \theta \) = angle of coordinate rotation
\( I_{\nu} \) = laser intensity
\( I_{\text{sat.}} \) = saturation intensity
\( A_{21} \) = Einstein coefficient for spontaneous emission (transition from state 2 to state 1)
\( B_{12} \) = Einstein coefficient for absorption
\( B_{21} \) = Einstein coefficient for stimulated emission
\( n_j(v, \nu) \) = population density of ions in state \( j \) moving with velocity \( v \) when the laser frequency is \( \nu \)
$b_{12}$ = absorption rate for ions in state 1 moving with velocity $v$ when the laser frequency is $\nu$

$b_{21}$ = stimulated emission rate for ions in state 2 moving with velocity $v$ when the laser frequency is $\nu$

$i(v, \nu)$ = line profile function for ions moving with velocity $v$

$L(\nu; \nu_0, v)$ = Lorentzian line profile function for ions moving with velocity $v$

$\gamma$ = Lorentzian line width parameter

$\nu_0$ = line center frequency

$n_g$ = neutral gas density

$\sigma$ = photon absorption cross section

$\nu_q$ = frequency for excited state quenching by ion-atom collisions

$\nu_e$ = electron-impact transition frequency

$n_e$ = electron density

$n_i(v)$ = total population density of ions with velocity $v$ in the two-level saturation model

$i_D(v)$ = Doppler line profile function

$g_j$ = statistical weight for state $j$

$h$ = Planck’s constant

$\Delta \nu_{FWHM}$ = line profile full-width at half-maximum in frequency space
I. Introduction

The 12.5 kW Hall Effect Rocket with Magnetic Shielding (HERMeS),\textsuperscript{1–3} under development by NASA Glenn Research Center (GRC) and Jet Propulsion Laboratory (JPL), is the central component of the Ion Propulsion System (IPS) for a Solar Electric Propulsion Technology Demonstration Mission (SEP-TDM)\textsuperscript{4} that will demonstrate high-power SEP technology and pave the way for future robotic and crewed missions to Mars and beyond. HERMeS operates at specific impulse up to 3000 s, with total thrust efficiency $\geq 60\%$ over a range of throttle levels, and its design operational lifetime exceeds 50,000 hours.\textsuperscript{2} This long lifetime, which represents a factor of 5–10 increase over the state-of-the-art in Hall thrusters prior to 2010, is possible thanks to the elimination of discharge channel erosion as a significant concern through the use of a magnetically shielded field topology.\textsuperscript{5–8}

As NASA moves toward more ambitious missions requiring high-power thrusters that must operate for years in space, life qualification through long-duration wear-testing in ground test facilities is becoming increasingly impractical. Fortunately, computational models of Hall thrusters have advanced to the point that accurate predictions of life limiting surface erosion rates can be achieved through simulations.\textsuperscript{9} HERMeS will be qualified for flight through a combination of wear testing and modeling, supported by additional short-duration experiments designed to elucidate the physics underlying life-limiting mechanisms and inform the models.

Modeling of the HERMeS thruster’s performance and operational lifetime is being carried out with JPL’s Hall2De,\textsuperscript{10–13} a code that solves the electron and ion fluid equations in two dimensions on a magnetic field-aligned mesh and includes a particle-in-cell (PIC) module for calculating erosion by high-energy ions. Hall2De captures most of the important physics relevant to magnetically shielded thruster operation, but like all other existing Hall thruster codes, it cannot yet independently predict the electrostatic potential profile in the acceleration zone, which depends on non-classical (anomalous) electron transport across the radial magnetic field. While recent progress on the anomalous transport problem\textsuperscript{9, 14–19} provides hope that fully self-consistent fluid simulations may soon be able to make accurate stand-alone calculations of the potential, at present experimental inputs are still needed to set the magnitude and spatial dependence of the anomalous collision frequency. The necessary data can be obtained using laser-induced fluorescence (LIF),\textsuperscript{20, 21} a powerful technique for non-invasively measuring the spatial dependence of the ion velocity distribution function (IVDF) in a plasma. Once the IVDF is known as a function of position, the electric field that accelerated the ions can be calculated analytically or inferred through comparisons with simulations.

Although it is also possible to measure the plasma potential as a function of position using an emissive probe, the insertion of a fast-scanning probe into a Hall thruster channel can cause the discharge to shift axially by up to 20\% of the channel length.\textsuperscript{22} Inaccuracies of this magnitude in the plasma location would lead to unacceptably large errors in the lifetime predictions produced by Hall2De; in particular, the rate of front pole erosion, which is expected to be one of the dominant life-limiting mechanisms in magnetically shielded thrusters, is strongly dependent on the axial location of the acceleration zone.\textsuperscript{12, 13} Therefore, minimally perturbative measurements of the axial ion velocity profile using LIF are a critical element of the HERMeS risk reduction and thruster life validation program.

This paper presents LIF measurements of the mean Xe II (singly ionized xenon) velocity in HERMeS Technology Demonstration Unit 2 (TDU-2)\textsuperscript{23} as a function of axial position along the channel centerline, and also at a number of off-centerline locations in the channel and near-plume. Raw LIF data along with curve fits are shown in order to provide a foundation for the mean velocity results and discuss important features such as the bimodal time-averaged IVDFs observed in the acceleration zone. Section II describes the vacuum facility and thruster configuration, LIF hardware setup, and data analysis procedures. Channel centerline ion velocity measurements are presented in Sec. III, and two-dimensional velocity measurements on and off the channel centerline are presented in Sec. IV. Finally, Appendix A provides a proof that the axial and radial mean velocities $v_z$ and $v_r$ can be calculated by a rotation of velocity components measured along other orthogonal axes in the $r$-$z$ plane, and Appendix B discusses LIF saturation and attempts to clarify some points that have caused confusion and disagreement in the literature.
II. Experimental Setup

A. Thruster and Facility

HERMeS TDU-2 is designed to operate at discharge voltages \( V_d \) ranging from 300–800 V, and discharge currents \( I_d \) as low as 6 A and as high as 31.3 A.\(^1\) The test campaign described here focused primarily on four designated Reference Firing Conditions at \( I_d = 20.83 \) A and \( V_d = 300, 400, 500, \) and 600 V. Across this range of throttle points, the nominal magnetic field strength \( (B_{\text{nom.}}) \) is constant, and the thruster can operate stably at fields up to 25% higher or lower than the nominal setting. The electron source for the discharge was a center-mounted lanthanum hexaboride (LaB\(_6\)) hollow cathode.\(^2\)

The tests were carried out in the JPL Owens facility, a \( \sim 3 \) m diameter, \( \sim 10 \) m long cryogenically-pumped vacuum chamber lined with graphite. The operating pressure measured by a xenon-calibrated Stabil ion gauge located at the thruster exit plane was below \( 1.3 \times 10^{-5} \) Torr during the collection of all data presented in this paper, except for cases when the pressure was intentionally increased to study facility effects. During these studies, xenon was injected toward the downstream graphite beam dump using an auxiliary gas feed located several meters downstream of the thruster.

The Hall thruster discharge power supply was a Magna-Power LXI 45 kW supply. An RLC filter was installed between the power supply and thruster.\(^2\) Discharge current oscillations were monitored on the anode and cathode lines with Pearson current transformers, and a high-speed differential voltage probe monitored the anode-to-cathode voltage. The thruster was operated exclusively in a “cathode-tied” configuration,\(^1\) with the thruster body electrically connected to cathode common and isolated from facility ground.

B. Background on LIF

Laser photons incident on a plasma will only be absorbed if their Doppler-shifted frequency in an atom or ion’s reference frame corresponds to the energy of an allowed bound electron transition. LIF\(^{20, 21, 25–27}\) exploits this fact to selectively interact with xenon ions moving with a particular velocity in the direction parallel to the laser propagation. Electrons bound to ions or atoms are excited from a lower energy level (often metastable) to a higher level, from which they spontaneously decay while emitting a photon. The intensity of this fluorescence, which in a typical non-resonant LIF scheme is detected at a wavelength differing from the laser wavelength, is proportional to the density of ions moving at the velocity probed by the laser (more precisely, it is proportional to the population density of atoms/ions moving with the correct velocity that are in the atomic state probed by the laser, which at a given location can generally be assumed to be proportional to the overall density of ions with that velocity). By scanning the laser through a range of wavelengths in order to interact with ions moving with different velocities, the ion velocity distribution function (IVDF) can be measured.

If the rest frame wavelength for a transition is \( \lambda_0 \) and the laser wavelength in the lab frame (in vacuum) is \( \lambda_{\text{laser}} \), the velocity of ions probed is

\[
v = c \left( \frac{\lambda_{\text{laser}}}{\lambda_0} - 1 \right),
\]

where \( c \) is the speed of light. Once the IVDF \( f_i(v) \) is known, the mean velocity \( u_i \) can be calculated from the first moment of the distribution:

\[
u = \int_{-\infty}^{\infty} v f_i(v) \, dv.
\]

In this study, we employed a common non-resonant LIF scheme driving the 834.953 nm transition from the Xe II \( 5p^4(3P_2)5d \, ^2[4]7/2 \) metastable state to the \( 5p^4(3P_2)6p \, ^2[3]5/2 \) state, with fluorescence collected at a vacuum wavelength of 542.06 nm (541.92 nm in air). Spatial resolution of \( \sim 2 \) mm was achieved by arranging for the collection optic line of sight (see Fig. 2) to intersect the laser beams in only a small volume. A side effect of this arrangement, which is standard for LIF, is that the signal-to-noise ratio of the measurement is intrinsically poor because the collection optic gathers light spontaneously radiated from a much larger volume of plasma. This difficulty is usually overcome by modulating the laser and using lock-in amplification to extract the desired LIF signal from the background light.
C. LIF Hardware

The JPL LIF setup uses a New Focus Velocity TLB-6716 tunable diode laser (15 mW max, 800–950 nm, 200 kHz line width, mode-hop-free tuning range \( \sim 80 \) GHz) driving a New Focus TA-7616 tapered amplifier (500 mW max, 825–855 nm). The beam was split along two paths on an optical table (\( \sim 66\%/33\% \) split) and coupled into 50 µm core, 0.22 numerical aperture (NA) optical fibers for transmission to the chamber. Mechanical choppers were used to modulate the beams along the two paths at 2 kHz and 3 kHz, respectively. An additional beam splitter was used to sample 10% of the beam for wavelength and power diagnostics. During LIF testing, \( \lambda_{\text{laser}} \) was continuously monitored using a Toptica WS/7 self-calibrating wavemeter with \( \pm 60 \) MHz accuracy.

As shown in Fig. 1, laser power was coupled into the chamber and carried by stainless-steel clad fibers to two injection optics oriented at 90 degree angles relative to one another and offset by 45 degrees from the thruster centerline. Each injection optic (see Fig. 2) consisted of a 25.4 mm diameter, 40 mm focal length plano-convex lens, with the fiber input position adjusted to optimize the beam focus at the LIF interrogation volume \( \sim 1 \) m from the optic. With the laser operating at full power, the time-averaged optical power delivered into the \( \sim 2 \) mm diameter interrogation spot was \( \sim 89 \) mW for the “East” line of sight and \( \sim 44 \) mW for the “West” line of sight.

The collection optic consisted of a pair of 75 mm diameter plano-convex lenses that collimated light collected from the plasma and focused it into a 600 µm core, 0.22 NA fiber. This optic was mounted above the thruster, as shown in Fig. 2(a), with its line of sight angled approximately 20 degrees from vertical in order to enable it to see into the discharge channel.

The two injection lines of sight and the collection line of sight were aligned to intersect at a single stationary point. During testing, the thruster was translated on a two-axis motion stage (see Fig. 1) to measure the IVDF at different locations. The motion stage positions were tracked by inductive linear encoders built into the stage assembly.

The injection optics were mounted on remotely controlled two-axis fine motion stages for in-situ adjustment of the alignment under vacuum. At the beginning of each test day, the thruster was warmed up for at least 1.5 hours, then it was shut down and re-alignment with low-power visible lasers was carried out while the thruster was still hot. By noting the motion stage encoder readings when the visible laser spots intersected various reference positions on the thruster face, we were able to calibrate the measurement location with an estimated accuracy of \( \pm 0.2 \) mm. This claim is supported by the high degree of reproducibility of velocity vs. position measurements taken on different days (see Fig. 4 for an example). At the end of each test day, optical alignment was checked again immediately after shutting down the thruster.

Plasma emission gathered by the collection optic was fiber-coupled to a photomultiplier tube (PMT) with an optical bandpass filter at the input. The PMT output was passed through a current amplifier and into two Stanford Research Systems 830 lock-in amplifiers, each set to extract signals at one of the optical chopper frequencies. The lock-in amplifier integration time was set to 300 ms. At each location probed in the
plasma, two 80-point wavelength scans were taken over a range spanning $\sim 0.09$ nm, with the laser operating in constant power mode. Each time the laser wavelength was changed, the data acquisition software would wait four lock-in amplifier integration times for the signal to stabilize before saving data.

D. LIF Data Analysis

1. Line Shifts and Broadening

Equation 1 is only strictly valid when all other sources of line shifts and broadening, including Zeeman splitting, hyperfine structure, saturation broadening, natural broadening, and the laser line width, are negligible compared to the Doppler width of features of interest in the IVDF. When any of these effects are non-negligible, at a given laser wavelength setting $\lambda_{\text{laser}}$, photons can be absorbed by metastable ions moving with a range of velocities, and the measured IVDF will be smeared out. Correcting for these distortions of the lineshape is most important when knowledge of the width of the IVDF is desired; symmetric broadenings will not affect the first moment of the distribution (Eq. 2), which is the quantity of interest in this study.

We will show in Appendix B that asymmetry in the hyperfine structure shifts the apparent mean velocity measured by LIF by $< 100$ m/s. Since this is much smaller than most of the mean velocities measured in this paper, this effect will be neglected in the analysis. For typical peak magnetic field strengths in a Hall thruster channel of a few hundred Gauss, Zeeman splitting of $\sigma$-polarized light can have an important effect on the apparent width of the IVDF in regions with relatively low ion temperature. However, the splitting is approximately symmetric, so it can be neglected in our analysis. The laser line width ($\sim 200$ kHz) and the natural broadening for the 834.953 nm transition ($\sim 20$ MHz) are also negligible.

Saturation broadening occurs when the laser intensity is high enough that the stimulated emission rate from the upper state of the LIF transition becomes comparable to the spontaneous emission rate, so the intensity of isotropic fluorescence emission no longer scales linearly with laser power. For a homogeneously broadened line (i.e., a line broadened by a mechanism such as natural broadening that affects all ions equally), saturation will occur at the line center before it occurs in the wings; as a result, the measured fluorescence intensity in the wings can continue to increase with laser intensity even as the line center amplitude asymptotes, leading to an additional artificial broadening of the line profile. Saturation studies carried out during the HERMeS TDU-2 LIF campaign, presented in Appendix B, showed that we were operating in a moderately saturated regime, which was beneficial for maximizing the signal-to-noise ratio of the LIF measurements. The detailed discussion in this appendix demonstrates that saturation was not problematic for the mean velocity calculations, and furthermore explains why saturation occurs at a lower laser intensity but has a smaller impact on the apparent width of LIF line profiles than has often been assumed in the Hall thruster literature. The key point is that saturation fundamentally affects the homogeneous lineshape; in the presence of dominant Doppler (inhomogeneous) broadening, its distorting effect is muted.
2. Data Analysis Procedure

After subtracting off the mean background light signal recorded at wavelengths near the limits of the laser scan, the LIF data from each line of sight were fit with a single or bi-Maxwellian function of the form:

\[
f_1(v) = A_1 \exp \left( -\left( \frac{v - v_1}{v_{Ti1}} \right)^2 \right)
\]

or

\[
f_2(v) = A_1 \exp \left( -\left( \frac{v - v_1}{v_{Ti1}} \right)^2 \right) + A_2 \exp \left( -\left( \frac{v - v_2}{v_{Ti2}} \right)^2 \right)
\]

The mean velocity along the injection beam direction was taken to be \( v_1 \) for a single Maxwellian fit, or a weighted average of \( v_1 \) and \( v_2 \) for a bi-Maxwellian fit:

\[
\langle v \rangle = \frac{A_1 v_{Ti1} v_1 + A_2 v_{Ti2} v_2}{A_1 v_{Ti1} + A_2 v_{Ti2}}
\]

(note that \( A v_{Ti} \) is proportional to the area under the Gaussian curve). This fitting procedure was preferred over a direct calculation of the mean velocity by numerical integration of Eq. 2 over the entire IVDF because it was less influenced by random scatter in the data in the line wings and therefore produced smoother plots of mean velocity vs. axial position. The form of the fitting functions in Eqs. 3 and 4 should not be taken to imply that the IVDF always consisted of one or two Maxwellian populations; rather, these were simply convenient functions to use, and the 6 fitting parameters \( (A_1, A_2, v_1, v_2, v_{Ti1}, v_{Ti2}) \) in Eq. 4 generally provided enough freedom to obtain a good fit to the data, except for a few cases of IVDFs measured near the channel edges and chamfers.

In order to prolong the life of the optics and to enable two-dimensional LIF data to be acquired within the discharge channel, the injection lines of sight were chosen to be 45 degrees offset from the axial and radial directions in the thruster coordinate system, as illustrated in Fig. 1. As a result, the full IVDF was only known along these two injection directions. However, the axial and radial components of the mean velocity could be calculated from the mean velocities along the East and West injection paths by a coordinate rotation:

\[
\begin{bmatrix}
\langle v_z \rangle \\
\langle v_r \rangle
\end{bmatrix} = \begin{bmatrix}
\cos(\pi/4) & \sin(\pi/4) \\
-\sin(\pi/4) & \cos(\pi/4)
\end{bmatrix} \begin{bmatrix}
\langle v_{east} \rangle \\
\langle v_{west} \rangle
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
-1 & 1
\end{bmatrix} \begin{bmatrix}
\langle v_{east} \rangle \\
\langle v_{west} \rangle
\end{bmatrix}.
\]

A proof of the general validity of this rotation procedure is provided in Appendix A. This appendix also shows that it is not necessarily correct to calculate the most probable velocity (i.e., the velocity at the peak of the distribution) along the axial and radial directions by applying a rotation transformation to most probable velocities measured along the diagonal East and West lines of sight.
Figure 4. Mean axial ion velocities along the channel centerline during $V_d = 300$ V, $I_d = 20.83$ A, $B = B_{nom}$, operation with the standard carbon inner pole cover installed, or with a thinner stainless steel pole cover in place.

Once the axial mean velocity $\langle v_z \rangle$ was known at a number of positions along the channel centerline, the plasma potential as a function of position could be estimated by assuming that ions traveling at the mean velocity were born at the anode potential, and applying conservation of energy:

$$\phi(z) = \phi_{anode} - \frac{m_i \langle v_z \rangle^2}{2e},$$

(7)

where $m_i = 2.19 \times 10^{-25}$ kg is the xenon ion mass and $e = 1.6 \times 10^{-19}$ C is the ion charge. More sophisticated procedures have been developed that use the full IVDF to calculate the potential profile and ionization rate; however, in this case we could not apply the 1D method of Pérez-Luna et al. along the channel centerline because the full IVDF along the axial direction was not measured, and it was not clear that the condition $f(x, v_{east}, v_{west}, t) = f(x, v_{east}, t) \times f(x, v_{west}, t)$ necessary for straightforward application of Spektor’s 2D method was satisfied in HERMeS. For practical purposes, the mean velocity measurements themselves were sufficient to determine the empirical anomalous collision frequency profile for Hall2De simulations. Plasma potentials estimated from Eq. 7 will be shown in Sec. III mainly for illustrative purposes.

III. Channel Centerline Results

A. Results with Nominal Magnetic Field and Background Pressure

1. Mean Velocities and Potentials

Figure 3(a) shows the mean axial velocities for Xe II ions measured by LIF as a function of position along the channel centerline for the $I_d = 20.83$ A, $V_d = 300–600$ V HERMeS operating conditions. Here and throughout the paper, distances are normalized to the discharge channel length $L_{channel}$, $z = 0$ is defined to be the axial location of the anode, and $r = 0$ is defined to be the thruster centerline. Fig. 3(b) shows the plasma potential inferred from the mean ion velocities using Eq. 7. It is notable that the potential profiles for the four different discharge voltages lie approximately on top of one another downstream of the $\phi = 300$ V point. At higher discharge voltages, ion acceleration began further and further upstream. There may have been a corresponding narrowing of the ionization zone, which begins upstream of the acceleration zone, because the higher electron temperature at high discharge voltages would have enabled most of the anode flow to be ionized in a narrower region.

In order to protect against pole piece erosion, which can be an important potential failure mechanism for magnetically shielded thrusters over the long lifetime required of HERMeS, the thruster uses carbon covers on the inner and outer front poles. While troubleshooting an outgassing issue at the beginning of the LIF test campaign, the inner pole cover was temporarily replaced by a stainless steel cover with less than half the thickness of the standard cover. LIF data was collected to confirm normal operation in this
Figure 5. IVDFs measured on the channel centerline at $V_d = 300$ V, $I_d = 20.83$ A, $B = B_{nom}$. a) Upstream of the acceleration zone. b) Within the acceleration zone. c) Downstream of the acceleration zone.

Figure 6. IVDFs measured on the channel centerline at $V_d = 400$ V, $I_d = 20.83$ A, $B = B_{nom}$. a) Upstream of the acceleration zone. b) Within the acceleration zone. c) Downstream of the acceleration zone.

Figure 7. IVDFs measured on the channel centerline at $V_d = 500$ V, $I_d = 20.83$ A, $B = B_{nom}$. a) Upstream of the acceleration zone. b) Within the acceleration zone. c) Downstream of the acceleration zone.

Figure 8. IVDFs measured on the channel centerline at $V_d = 600$ V, $I_d = 20.83$ A, $B = B_{nom}$. a) Upstream of the acceleration zone. b) Within the acceleration zone. c) Downstream of the acceleration zone.
configuration prior to re-installing the carbon pole cover. Ion velocities measured at the 300 V, 20.83 A operating condition with the two different pole covers are compared in Fig. 4. These results indicate that the pole cover thickness had a negligible impact on the plasma characteristics along the channel centerline. Furthermore, the close agreement between the two datasets taken several days apart (with the chamber vented in between) is evidence for the very good reproducibility of the LIF alignment procedures.

2. Ion Velocity Distribution Functions

Figures 5–8 show typical IVDFs measured within the discharge channel upstream of the acceleration zone, within the acceleration zone, and downstream of the acceleration zone in the near plume. The raw data (with the residual background light signal subtracted off) is plotted as dots, and single or bi-Maxwellian fits are overlaid. “East” and “West” label the two laser injection directions (see Fig. 1).

Upstream of the acceleration zone, the IVDFs were narrow, with low mean velocity. Plots such as Fig. 3(a) show that there was an upstream position beyond which the mean axial ion velocity was negative (i.e., directed back toward the anode). This detail is also captured in Hall2De simulations.

Within the acceleration zone, the mean ion velocity increased and the time-averaged distribution became broadened. As the discharge voltage was increased from 400–600 V, there was increasing bifurcation of the IVDF. This behavior likely resulted from oscillations of the acceleration zone position—it will be discussed in detail in Sec. III.A.3.

Downstream of the acceleration zone, the velocity distributions remained relatively broad, and while
they no longer had two distinct peaks, they did not appear to be Maxwellian. In all but the $V_d = 600$ V case, there was a clear excess of high-velocity ions on the right-hand side of the distribution. This feature is thought to be due to kinematic distortion,25 a re-shaping of the distribution function that occurs when collisionless charged particles are accelerated through a potential difference. “Kinematic compression” is also expected to narrow the distribution function in the ideal case, but in Hall thrusters there are other competing broadening effects including overlap of the ionization and acceleration zones, time-averaging of IVDFs that oscillate in space, and ion heating from wave-particle interactions. Note that another possible source of fast Xe$^+$ ions, charge exchange processes involving double ions (Xe$^{2+}$), cannot be the main source of the high-energy tail on the measured IVDFs because the mean free paths for the Xe$^{2+}$ + Xe$^+$ → Xe$^+$ + Xe$^{2+}$ and Xe$^{2+}$ + Xe → Xe$^+$ + Xe$^+$ processes are greater than one meter in the HERMeS near plume,32,33 while the observed distortion of the IVDFs occurred over a distance of only a few centimeters.

### 3. Evidence for Oscillations in the Acceleration Zone Position

In Figs. 7(b) and 8(b), two peaks are visible in the IVDFs. Given that ionization happens over a continuous range of axial positions and all ions see the same electric field at a given position, there is no reason to expect that two populations with such a wide velocity spread could develop. A more likely explanation for the data is that the acceleration profile was oscillating in space, so that the mean axial velocity $z/L_{\text{channel}} = 0.96$ was $\sim 3$ km/s at one phase of the oscillation and $>15$ km/s at another phase. Since the LIF data was averaged over a long timescale on the order of the lock-in amplifier integration time constant (300 ms),

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**Figure 11.** Discharge voltage and current oscillations at $V_d = 500$ V, $I_d = 20.83$ A, $B = B_{\text{nom}}$. a) Time domain plot of voltage and current. b) Frequency domain plot of anode current.

**Figure 12.** Discharge voltage and current oscillations at $V_d = 600$ V, $I_d = 20.83$ A, $B = B_{\text{nom}}$. a) Time domain plot of voltage and current. b) Frequency domain plot of anode current.
Figure 13. Simulated LIF data for a drifting Maxwellian IVDF \((T_i = 1 \text{ eV})\) with oscillating mean velocity given by Eq. 9. a) \(\Delta v_0 = 0\). b) \(\Delta v_0 = 1 \text{ km/s}\). c) \(\Delta v_0 = 2 \text{ km/s}\). d) \(\Delta v_0 = 5 \text{ km/s}\).

The measured IVDFs were smeared out over the oscillations, with apparent peaks at the extremes due to the nearly flat slope of \(\cos(\omega t)\) near \(\omega t = 0\) and \(\omega t = \pi\). Motion of the ionization and acceleration zones during Hall thruster breathing mode oscillations has previously been observed in several time-resolved LIF studies.\(^{34-36}\)

This interpretation is supported by Figs. 9–12, which show that large amplitude, coherent oscillations in the discharge current and voltage were present at the 500–600 V operating conditions where strongly bimodal time-averaged IVDFs were measured. On the other hand, at the 300–400 V operating conditions, the amplitude of the discharge oscillations was much smaller, and they had a broad frequency spectrum rather than a well-defined fundamental frequency. Time-averaging over these smaller oscillations probably led to the less pronounced two-peak structure in the 400 V data shown in Fig. 6(b), and they may have also caused some broadening of the IVDF at 300 V (Fig. 5(b)).

In order to provide further support for the hypothesis that the measured bimodal IVDFs arose due to time averaging over oscillations, simple simulations of this time-averaging were carried out for a drifting Maxwellian velocity distribution with an oscillating mean velocity \(v_0(t)\):

\[
f_i(v) = \frac{A}{v_0(t)} \exp \left( -\frac{m_i (v - v_0)^2}{2k_BT_i} \right),
\]

\[
v_0(t) = v_0(t = 0) + \Delta v_0 \sin(\omega t).
\]

\(f_i(v)\) was assumed to be proportional to \(1/v_0(t)\) in order to account for the reduction in ion density at higher mean velocities due to particle flux conservation. Examples of simulated time-averaged LIF line profiles in the acceleration zone are shown in Fig. 13. These plots bear qualitative resemblance to the data in Figs. 5(b)–8(b). Better agreement could likely be obtained if creation of new ions in the acceleration zone were accounted for.
To estimate how far the acceleration zone moved during the breathing mode oscillation, in Fig. 14 we have separately plotted the mean velocities of the “slow” and “fast” ion populations that appeared to exist in the time-averaged IVDFs in Figs. 5–8(b). The offset between the curves implies that the acceleration zone was oscillating \( \sim 5\% \) of the channel length in either direction from its mean location at \( V_d = 500–600 \) V, a smaller distance at \( V_d = 400 \) V, and a negligible amount at \( V_d = 300 \) V.

The oscillations in the acceleration zone position are of particular importance for HERMeS because they increase the pole erosion rates and thus can affect thruster lifetime. Hall2De simulations\(^\text{13}\) of the 600 V operating condition without breathing mode oscillations (which do not always arise spontaneously in the code) predicted an inner pole cover erosion rate that was approximately 5 times lower than the measured rate from recent wear tests at NASA GRC\(^\text{37}\) and surface layer activation measurements at JPL\(^\text{38}\). When discharge current oscillations with frequency and amplitude matching the experiments were forced by varying the anomalous collision frequency profile in the simulation, the predicted inner pole cover erosion rate increased to approximately match the data.

B. Results with Varying Magnetic Field Strength

Figure 15 shows how the acceleration zone shifted as the magnetic field strength was varied between 75\% and 125\% of its nominal value. At the \( V_d = 300 \) V operating condition, the acceleration profile was flatter at \( B = 1.25B_{\text{nom}} \) than at lower magnetic fields, but this trend was not observed at higher discharge voltages. At all voltages except 400 V, the acceleration zone moved upstream at higher magnetic field strengths.

Notably, the inner pole cover erosion rate at the 600 V operating condition measured by surface layer activation at JPL\(^\text{38}\) was lowest at \( B = 0.75B_{\text{nom}} \) and highest at \( B = 1.25B_{\text{nom}} \). Generally we would expect more pole erosion as the mean location of the acceleration zone moves downstream, but some other mechanism...
must have been the dominant driver of the trend in this case. Evidence from Hall2De simulations suggests that higher magnetic strengths led to higher resistivity between the cathode plume and the thruster plume, which increased the plume potential and led to higher energies for ions impacting the pole covers. This conclusion is consistent with the trend in the mean velocities shown in Fig. 15 at the furthest downstream point interrogated: \( u_i \) decreased with increasing magnetic field strength, implying that the potential drop between the ionization zone and the plume was smaller at large \( B \).

C. Results with Elevated Facility Background Pressure

Background pressure effect studies were carried out in order to assess the relevance of LIF measurements carried out at \( \sim 10 \) µTorr in the JPL Owens facility for Hall2De simulations of the HERMeS thruster lifetime in flight. Fig. 16 compares the mean ion velocity profiles at the nominal xenon background pressure and twice the nominal pressure for each discharge voltage from 300–600 V. At elevated background pressure, the acceleration zone moved upstream slightly, but the shift was less than 5% of the channel length at all discharge voltages. These results imply that the acceleration zone might shift further downstream under space vacuum, potentially increasing pole erosion above the rates measured in ground tests and predicted by simulations based on those tests. Fortunately, the magnitude of this effect appears to be small.

There is some risk in attempting to extrapolate our results to space conditions, since Hall thruster performance does not always scale linearly with pressure at low pressures; however, this nonlinearity has been found to be minimal in magnetically shielded thrusters with center-mounted hollow cathodes. Additional LIF measurements on HERMeS at pressures as low as \( \sim 4 \) µTorr will be carried out at NASA GRC in the future.
D. Results with Enhanced Discharge Voltage Ripple

Studies of the dependence of the time-averaged acceleration zone location on the magnitude of discharge voltage oscillations were carried out in order to gain insight into the importance (or lack thereof) of voltage ripple limits levied on the power processing unit (PPU) for IPS. Auxiliary coils were added in series with the discharge filter in order to increase the harness inductance from the nominal value of ∼8 μH to either ∼16 μH or ∼32 μH. With the modified versions of the discharge circuit, the peak-to-peak breathing mode oscillations in the discharge voltage at ⟨V_d⟩ = 600 V, ⟨I_d⟩ = 20.83 A, and B = B_{nom.} increased from 82 V to 124 V and 198 V, respectively. Mean ion velocities measured under these conditions are presented in Fig. 17. The acceleration zone location varied by only 2–3% of the channel length across the three voltage ripple cases, suggesting that the voltage oscillation amplitude had very little effect on the time-averaged discharge properties.

IV. Two-dimensional Ion Velocity Maps

LIF data were collected at the V_d = 300, 400, and 600 V operating conditions (I_d = 20.83 A, B = B_{nom.}) across a grid of points spanning the outer half of the channel and near plume, and near both the inner and outer chamfers at the downstream edge of the channel. The mean velocity vectors and a selection of the off-centerline IVDFs are shown in Figs. 18–20. Figures 21(a)–(c) zoom in on the channel to show the mean flow directions of the slow ions upstream of the acceleration zone.

Some of the time-averaged IVDFs far from the channel centerline were highly non-Maxwellian, with as many as three peaks, so fits using Eq. 4 could not always capture the full complexity of the IVDF. The fits were sufficient, however, for a reasonable estimate of the mean velocity in each injection direction. The LIF...
data taken near the inner and outer chamfers at $V_d = 300$–400 V had poor signal-to-noise ratio, but in all but a few cases the most probable or mean velocity could still be estimated, even if it was not possible to reliably fit the width of the distribution. The orientation of the mean velocity vectors at points adjacent to the chamfers was approximately parallel to the boron nitride surface.

A critical feature of the data in Figs. 18–20 is that the radial divergence of the ion trajectories increased as the discharge voltage was decreased. This point is illustrated more explicitly in Fig. 22, which overlays the velocity vectors from the three discharge voltages studied. Ions born at high potentials near the edge of the beam are suspected to be the main source of pole erosion in magnetically shielded thrusters, and the electric fields that produced the observed ion velocity divergence at lower discharge voltages (particularly $V_d = 300$ V) would tend to drive a higher flux of ions toward the pole faces. Thus the LIF results are consistent with the high pole erosion rate observed at the $V_d = 300$ V, $I_d = 20.83$ A operating condition during surface layer activation experiments at JPL and short duration wear tests at NASA GRC.

The pole erosion rate was initially expected to be highest at the full power 600 V, 20.83 A operating condition; consequently, the first wear test of HERMeS TDU-1 was carried out almost exclusively at this condition. As further testing has revealed similar or higher erosion rates during lower power operation, ongoing modeling and experimental work has sought to explain this erosion. The particularly high erosion rate at 300 V, 20.83 A has proven difficult to capture in simulations (recall that the large-amplitude discharge current oscillations accompanied by acceleration zone movement that have been found to contribute significantly to pole erosion at $V_d = 600$ V were not present at $V_d = 300$ V). At the time when the LIF measurements were taken, Hall2De could not reproduce the highly divergent mean ion velocity vectors shown in Fig. 18. Efforts to resolve this discrepancy have focused on possible factors such as changes in the way that the anomalous collision frequency varies along magnetic field lines that could alter the potential distribution in the channel enough to accelerate ions along the observed trajectories.

V. Conclusion

In addition to enabling the spatial dependence of the anomalous collision frequency to be fixed empirically in Hall2De simulations of the HERMeS thruster, the ion velocity measurements presented in this paper place constraints on any self-consistent model of anomalous transport, which must be able to reproduce the acceleration zone location and the shape of the potential gradient over the range of operating conditions studied. The results presented in Sec. IV represent the most detailed information ever obtained about ion trajectories away from the channel centerline in a magnetically shielded thruster. These data have already provided valuable information for model validation, and they may ultimately lead to the discovery of new physics that is important near the acceleration zone and must be included in numerical models of Hall thrusters. Further measurements to capture 2D ion velocity data at additional operating conditions (such as $V_d = 500$ V, $I_d = 20.83$ A) and extend the maps in Figs. 18–22 to cover the inner half of the channel and near plume are planned for an upcoming test campaign. We also plan to carry out time-resolved
Figure 18. Mean velocity vectors across the outer half of the channel and near plume for the $V_d = 300$ V, $I_d = 20.83$ A, $B = B_{nom}$. operating condition. Raw IVDFs and curve fits are shown for the circled locations.
Figure 19. Mean velocity vectors across the outer half of the channel and near plume for the $V_d = 400$ V, $I_d = 20.83$ A, $B = B_{nom}$. operating condition. Raw IVDFs and curve fits are shown for the circled locations.
Figure 20. Mean velocity vectors across the outer half of the channel and near plume for the $V_d = 600$ V, $I_d = 20.83$ A, $B = B_{nom}$. operating condition. Raw IVDFs and curve fits are shown for the circled locations.
LIF measurements using the transfer-function averaging technique\textsuperscript{36,43} to confirm our interpretation of the bimodal time-averaged IVDFs shown in Figs. 6–8 and determine the detailed time dependence of the acceleration zone position, which will enable more accurate simulations of the pole erosion rates at high discharge voltages.

**Appendix A: Rotation of the Mean Velocity and Most Probable Velocity**

In this paper, mean axial and radial ion velocities were determined by using LIF to measure the mean velocities along two orthogonal injection beam directions offset 45 degrees from the axial and radial directions, and then rotating the coordinate system to find the components of the velocity vector in the $z$ and $r$ directions (see Sec. II.D.2 and Fig. 1). While measurements of the IVDF along two lines of sight are not sufficient to unambiguously determine the full IVDF along all other directions in the $r$-$z$ plane, we will prove in this appendix that the mean velocity rotation procedure is rigorously valid.

Consider a population of $N$ ions, each with some velocity $v_j = v_{xj}\hat{x} + v_{yj}\hat{y}$. The mean velocities in the $\hat{x}$ and $\hat{y}$ directions for the ensemble of particles are:

\begin{align}
\langle v_x \rangle &= \frac{1}{N} \sum_{j=1}^{N} v_{xj} \\
\langle v_y \rangle &= \frac{1}{N} \sum_{j=1}^{N} v_{yj}.
\end{align}

If we define a new coordinate system with axes $x'$ and $y'$, related to the original coordinate system by a
Figure 22. Mean velocity vectors from Figs. 18–20, overlaid on a single plot to show the trend in ion trajectory divergence versus discharge voltage.

counterclockwise rotation through an angle \( \theta \), then the rotated velocity vectors of the individual particles are:

\[
\begin{bmatrix}
v'_{xj} \\
v'_{yj}
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
v_{xj} \\
v_{yj}
\end{bmatrix}
\]

(12)

The mean velocity components in the new coordinate system are:

\[
\langle v'_{x} \rangle = \frac{1}{N} \sum_{j=1}^{N} v'_{xj} = \frac{1}{N} \sum_{j=1}^{N} (v_{xj} \cos \theta + v_{yj} \sin \theta)
\]

(13)

\[
= \cos \theta \left( \frac{1}{N} \sum_{j=1}^{N} v_{xj} \right) + \sin \theta \left( \frac{1}{N} \sum_{j=1}^{N} v_{yj} \right)
\]

(14)

\[
\langle v'_{y} \rangle = \frac{1}{N} \sum_{j=1}^{N} v'_{yj} = \frac{1}{N} \sum_{j=1}^{N} (-v_{xj} \sin \theta + v_{yj} \cos \theta)
\]

(16)

\[
= -\sin \theta \left( \frac{1}{N} \sum_{j=1}^{N} v_{xj} \right) + \cos \theta \left( \frac{1}{N} \sum_{j=1}^{N} v_{yj} \right)
\]

(17)

\[
= -\sin \theta \langle v_x \rangle + \cos \theta \langle v_y \rangle.
\]

(18)

So we can calculate the mean velocity components in the rotated coordinate system by applying a rotation matrix to the mean velocity vector measured in the original coordinate system.

This proof may seem intuitively obvious. However, note that a similar rotation procedure cannot be used for the most probable velocity (the velocity at the peak of the IVDF). This can be demonstrated with a simple counterexample. Consider a plasma in which there are only three ions, with velocity components along the East and West directions (oriented at 45 degree angles with respect to the axial and radial directions as shown in Fig. 1) and along the axial and radial directions given in Table 1. In this example, the most probable velocity along the East direction is 0. The most probable velocity along the West direction is also
Table 1. Three-particle example demonstrating how applying a coordinate rotation to the most probable ion velocity can lead to incorrect results.

<table>
<thead>
<tr>
<th>Ion #1</th>
<th>Ion #2</th>
<th>Ion #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{east}$</td>
<td>$v_{west}$</td>
<td>$v_z$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$1/\sqrt{2}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$1/\sqrt{2}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

0. Rotating this “most probable velocity vector”, we would find erroneously that the most probable axial velocity is 0. In fact, the most probable axial velocity is $1/\sqrt{2}$.

Appendix B: LIF Saturation

As discussed in Sec. 1, at relatively low laser intensities the fluorescence signal from LIF is linearly proportional to the laser power, but at higher intensities, as the upper atomic level population density becomes comparable to the lower level (target state) population density and stimulated emission starts to compete with spontaneous emission, the transition becomes saturated. Saturation can lead to spurious broadening of the measured line shape, and because it is difficult to quantify and correct for this distortion, there is a feeling among many researchers that operating in a saturated regime should be avoided at all costs. However, using high laser power is beneficial for the LIF signal-to-noise ratio and may enable useful LIF measurements to be taken in low-density or cool regions of the plasma where an impractically long integration time would otherwise be required.

Over the last two decades, since LIF became a popular diagnostic for electric propulsion research, there has been some disagreement and confusion in published papers and PhD theses about LIF saturation in Hall thruster plasmas. While saturation intensities $I_{sat.}$ on the order of 0.1 mW/mm$^2$ for LIF transitions in xenon and other noble gases have been calculated in the plasma physics and electric propulsion literature, we have also seen estimates in the range 10–100 mW/mm$^2$ for Hall thrusters. The discrepancy arises based on whether one assumes the saturation intensity is proportional on the homogeneous line width (e.g., due to natural broadening), or the inhomogeneous line width due to Doppler broadening, which depends on the ion or gas temperature. We will show below that the homogeneous line width is the correct quantity to use. Other misleading statements that have appeared in articles and theses on Hall thruster include:

- The maximum possible LIF signal level at a given wavelength is twice the signal level obtained when the laser intensity is $I_\nu = I_{sat.}$.
- When $I_\nu \gg I_{sat.}$, the measured IVDF will always be significantly distorted.
- Strong transitions (i.e., transitions with large Einstein coefficient $A_{21}$) are easier to saturate than weak transitions.

The analysis below aims to clear up some of these misconceptions. We should note that none of this material is completely new—related discussions have appeared in various textbooks and articles on photonics and plasma diagnostics, and many Hall thruster publications have presented derivations similar to those that follow. Nevertheless, in light of the discrepancies in the details of these published derivations and the frequent misinterpretations of the results, we feel that there is value in revisiting the topic.

A. Two-Level Model of LIF Saturation in Hall Thrusters

We will analyze LIF saturation using a simple two-level atomic model in which only transitions between the lower metastable state (denoted by a subscript 1) and the upper state (denoted by a subscript 2) of the transition driven by the laser are considered. Four-level models that include the ground state and the final state following fluorescent emission have been published, but the two-level model is sufficient for deriving the basic scaling relationships, and its simplicity is advantageous for avoiding confusion. Furthermore, the results are independent of the electron temperature and density, which would no longer be the case for more complex models that account for collisional transitions between multiple states.
Define $n_1(v, \nu)$ and $n_2(v, \nu)$ to be the population densities of ions in state 1 and state 2 moving with velocity between $v$ and $(v + dv)$, when the laser frequency is $\nu$. We will solve rate equations to find the steady state population densities for each infinitesimal velocity range separately, and then integrate across the Doppler-broadened line profile to find the total upper level population density $n_2(\nu)$, which is proportional to the LIF signal strength. This procedure is valid because electronic transitions within the ion, whether spontaneous or driven by the laser, can never change its velocity, so the total population density of ions with velocity between $v$ and $(v + dv)$ is constant. The steady state rate equations are

$$\frac{d}{dt} (n_1(v, \nu)) = -b_{12}(v, \nu) n_1(v, \nu) + (b_{21}(v, \nu) + A_{21}) n_2(v, \nu) = 0$$
$$\frac{d}{dt} (n_2(v, \nu)) = b_{12}(v, \nu) n_1(v, \nu) - (b_{21}(v, \nu) + A_{21}) n_2(v, \nu) = 0.$$  

Here $A_{21}$ is the spontaneous transition rate from state 2 to state 1, $b_{12}(v, \nu)$ is the excitation rate due to absorption of laser photons, and $b_{21}(v, \nu)$ is the stimulated emission rate driven by the laser. These quantities are related to the Einstein coefficients $B_{12}$ and $B_{21}$ by

$$b_{12}(v, \nu) = \frac{B_{12} I_\nu i(v, \nu)}{c},$$
$$b_{21}(v, \nu) = \frac{B_{21} I_\nu i(v, \nu)}{c}.$$  

$I_\nu$ is the laser intensity (power per unit area) when it is tuned to frequency $\nu$, and $i(v, \nu)$ is the line profile function. We will assume that the laser line width is negligible compared to the natural line width (see Sec. II.D.1 for numerical estimates relevant to our setup), and for now we will assume that natural broadening is the only homogeneous broadening mechanism (hyperfine structure effects will be discussed in the next section). The line profile is then a Lorentzian function:

$$i(v, \nu) = \frac{\gamma/4\pi^2}{\left(\nu - \nu_0\right)^2 + (\gamma/4\pi)^2} \equiv L(\nu; \nu_0, \gamma).$$  

where $\nu_0$ is the rest frame line center frequency, $\nu/\left(1 - v/c\right)$ is the photon frequency seen by the ion ($v$ is defined to be positive for ions moving toward the laser), and $\gamma \equiv \sum_j A_{2j}$ for transitions with a metastable lower level.

In Eqs. 19 and 20, we have neglected collisional excitation and de-excitation processes. Unlike in gases near atmospheric pressure, quenching by ion-atom collisions should be of negligible importance in Hall thruster plasmas: assuming the ion velocity is $v \sim 10 \text{ km/s}$, the neutral density is $n_n \sim 10^{19} \text{ m}^{-3}$, and the collision cross section is $\sigma \sim 10^{-19} \text{ m}^2$, the quenching frequency would be $\nu_q = n_n \sigma v \sim 10^4 \text{ s}^{-1}$, which is much lower than the spontaneous de-excitation rate ($A_{21} > 10^6 \text{ s}^{-1}$) for transitions of interest for LIF. A typical rate coefficient for electron-impact transitions in noble gases is $\langle \sigma v \rangle = 10^{-13} \text{ m}^3/\text{s}$, so for an electron density $n_e \sim 10^{18} \text{ m}^{-3}$, the transition rate is $\nu_e \sim 10^5 \text{ s}$; thus neglecting these processes should also introduce only modest errors in the calculation. Given that background emission at the fluorescence wavelength dominates over the LIF signal prior to lock-in amplification, it may seem surprising that we can obtain a reasonable result while neglecting electron-impact excitation from state 1 to state 2. However, the strong background signal arises primarily because the LIF interrogation volume is only a small fraction of the total volume of plasma viewed by the LIF collection optic, and also because for a highly Doppler-broadened line profile, laser photons at a given wavelength can only interact with a small fraction of the ions within the interrogation volume. For these ions that have the correct velocity to interact with the laser beam, laser-driven excitations will dominate over electron-impact excitations in Hall thrusters whenever the laser intensity is an appreciable fraction of the saturation intensity.

Solving Eq. 19 or 20 yields

$$n_2(v, \nu) = \left(\frac{b_{12}(v, \nu)}{b_{21}(v, \nu) + A_{21}}\right) n_1(v, \nu).$$  

Defining $n_t(v) = n_1(v, \nu) + n_2(v, \nu)$ to be the total population density in states 1 and 2 with velocity between
Figure 23. Saturation calculations with hyperfine structure neglected. a) Simulated LIF signal at the peak of the IVDF as a function of laser intensity. b) Simulated LIF data showing the effect of saturation for a Maxwellian IVDF with $u_i = 10$ km/s and $T_i = 0.1$ eV. The curves all have the same normalization (unity area) in order to illustrate the impact of saturation on the line profile width. c) Same as (b) for $T_i = 1$ eV. d) Fractional error in the ion temperature inferred from the full-width at half-maximum (FWHM) of the simulated profile. The legend lists the correct $T_i$ values for the undistorted IVDF.

$v$ and $(v + dv)$ (which is a constant in the two-level model), this can be re-written as:

$$
n_2(v, \nu) = \frac{b_{12}(v, \nu)}{b_{21}(v, \nu) + A_{21}} \ n_t(v) = \frac{b_{12}(v, \nu)}{b_{21}(v, \nu) + A_{21} + b_{12}(v, \nu)} \ n_t(v). \tag{25}
$$

Note that $n_t(v)$ is proportional to the Doppler line profile function $i_D(v)$.

Substituting for $b_{12}(v, \nu)$ and $b_{21}(v, \nu)$ using Eqs. 21 and 22, this becomes

$$
n_2(v, \nu) = \left( \frac{B_{12}L_{\nu}L(v; \nu_0, v)}{B_{21}L_{\nu}L(v; \nu_0, v) + cA_{21} + B_{12}L_{\nu}L(v; \nu_0, v)} \right) \ n_t(v). \tag{26}
$$

When $B_{21}$ and $B_{12}$ are defined in terms of the radiation energy density (consistent with the convention adopted in Eqs. 21 and 22), the relationships between the Einstein coefficients18 are

$$
A_{21} = \frac{8\pi h \nu_0^3}{c^3} B_{21} \tag{27}
$$

$$
B_{12} = \frac{g_2}{g_1} B_{21}, \tag{28}
$$

where $h$ is Planck’s constant and $g_2$ and $g_1$ are the statistical weights of the upper and lower states, respec-
Figure 24. Simulated LIF signal for ions with $T_i = 0$ and $u_i = 0$ showing hyperfine structure and natural broadening. Note that there is an additional weak line component at $v = 3.4$ km/s that is not shown on the plot but was included in our model.

Figure 23 graphically illustrates the effects of LIF saturation predicted by Eqs. 23, 30, 31, and 32. Panel (a) shows the LIF signal at the peak of the distribution, as a function of laser intensity. At all values of the ion temperature, the signal increases linearly with $I_\nu$ for $I_\nu < I_{\text{sat}}$, and more slowly than linearly for $I_\nu > I_{\text{sat}}$. However, by continuing to increase the laser intensity above $I_{\text{sat}}$, it is possible to further increase the LIF signal by orders of magnitude above the value at the saturation threshold, particularly when the ion temperature is high. This result may seem surprising given that Eq. 30 implies that the signal contribution from the Lorentzian line center cannot exceed twice its value when $I_\nu = I_{\text{sat}}$. However, as $I_\nu$ is increased above $I_{\text{sat}}$, the underlying Lorentzian natural line profile is broadened by saturation, laser photons at the frequency $\nu$ corresponding to the peak of the LIF profile can interact with ions spanning a much greater range of velocities, enabling the LIF signal to continue to increase until the saturation-broadened Lorentzian line width approaches the Doppler profile width.

The fluorescence signal when the laser is tuned to frequency $\nu$ is proportional to the total upper level population including ions moving at all velocities:

$$n_2(\nu) = \int_{-\infty}^{\infty} n_2(v, \nu) \, dv.$$  \hspace{1cm} (31)

We define the saturation intensity to be the value of the second term in the denominator in Eq. 30 when $\nu/\left(1 - \frac{v}{c}\right) = \nu_0$ (the peak of the Lorentzian function defined in Eq. 23):

$$I_{\text{sat}} \equiv \left(\frac{g_1}{g_1 + g_2}\right) \left(\frac{2\pi \gamma \hbar \nu_0^3}{c^2}\right) = \left(\frac{g_1}{g_1 + g_2}\right) \left(\frac{2\pi \gamma \hbar c}{\lambda_0^3}\right).$$  \hspace{1cm} (32)

The $\gamma$ in the numerator means that the saturation intensity is proportional to $A_{21}$ (more precisely, it is the total spontaneous decay rate of the upper state into all other states that matters), so strong transitions are harder to saturate than weak transitions. For the Xe II 834.953 nm transition, \textsuperscript{49} using $g_1 = 8$ and $g_2 = 6$ and estimating $\gamma \approx 10^8$ s$^{-1}$, we find $I_{\text{sat}} \approx 0.12$ mW/mm$^2$. When $I_\nu \ll I_{\text{sat}}$, $n_2(\nu, \nu)$ scales linearly with $I_\nu$ for every infinitesimal velocity interval, so $n_2(\nu)$ (and therefore the LIF signal magnitude) also scales linearly with laser intensity. When $I_\nu \gtrsim I_{\text{sat}}$, the scaling of LIF signal with laser intensity becomes nonlinear.
Panels (b) and (c) of Fig. 23 show simulated LIF line profiles for Maxwellian plasmas with $T_i = 0.1$ eV and $T_i = 1$ eV, respectively, for several ratios of $I_\nu/I_{sat}$. While Eq. 31 gives the measured line profile as a function of laser frequency, the x-axis variable in the plots has been converted back to velocity using the Doppler shift relationship (see Eq. 1). Relatively little distortion of the IVDF is observed for laser intensities up to 100 times the saturation intensity—it is only for very high values of $I_\nu/I_{sat}$ that the line profile becomes noticeably broadened. This observation is quantified in panel (d), which shows the relative error in the $T_i$ calculated from the simulated LIF data using the full-width at half-maximum $\Delta \nu_{FWHM}$ of the profile:

$$\Delta \nu_{FWHM} = \nu_0 \sqrt{\frac{8 \ln 2 k_B T_i}{m_e c^2}}. \tag{33}$$

When the true ion temperature is 0.1 eV, the measurement will overestimate $T_i$ by 10% when $I_\nu/I_{sat} \approx 18$. On the other hand, if the actual $T_i$ is 10 eV, then $T_i$ can be measured with $<10\%$ error as long as $I_\nu/I_{sat} \lesssim 2000$.

B. Hyperfine Structure Effects

The previous section assumed that the natural lineshape consisted of a single Lorentzian profile, which is a significant oversimplification for xenon.\textsuperscript{25,26} Xenon has 9 stable isotopes, each of which have slightly different level energies. The levels in Xe-129 and Xe-131, which have an odd number of neutrons and non-zero nuclear spin, are further split by coupling between the electron angular momentum and the nuclear spin. These two isotopes produce a total of 12 nuclear spin components for the Xe II 834.953 nm line.\textsuperscript{50} Both the isotopic
Figure 26. Effect of saturation on the mean velocity calculation. a) Error in mean velocity for drifting Maxwellian IVDFs with $T_i = 1$ eV and $u_i = 1$ km/s or $u_i = 10$ km/s, and also for a calculation that assumed that the “West” data show in Fig. 7(c) represents the true IVDF. b) Simulated LIF data showing the effect of saturation and hyperfine structure, assuming the “West” data in Fig. 7(c) represents the true IVDF.

and nuclear spin components are very closely grouped for this line, so direct measurements of the hyperfine structure are very difficult. The most detailed results to date have been obtained through measurements and modeling by Pawelec et al.,\textsuperscript{50} we will use the component frequencies and relative amplitudes shown in the inset of Figure 8 of their paper to explore the effect of hyperfine structure on saturation. This figure shows 3 non-negligible nuclear spin components for Xe-129, 7 non-negligible nuclear spin components for Xe-131, and additional non-negligible contributions from 4 even isotopes.

We begin by noting that isotopic splitting is an inhomogeneous line splitting mechanism. We may group ions by their mass and calculate the LIF transition upper level population density $n_2(\nu)$ for each isotope individually, as we did for velocity in the previous section, and then add up these population densities for the different isotopes to get the total $n_2(\nu)$, which is proportional to the detected LIF signal. On the other hand, for ions of a given isotope, nuclear spin splitting is a homogeneous splitting mechanism, affecting all ions equally. Therefore, in our saturation calculation we should replace the single-peaked natural line profile function for these isotopes by a profile function that includes multiple Lorentzian peaks corresponding to each nuclear spin component.

Figure 24 shows the calculated LIF signal for a case with zero ion temperature, to illustrate the hyperfine structure we have assumed. The effect of saturation broadening for finite temperature ions with hyperfine structure accounted for is shown in Fig. 25. The results are only moderately different from Fig. 23. Once again, the maximum achievable LIF signal is several orders of magnitude higher than the signal level when $I_\nu = I_{sat}$. The hyperfine splittings introduce some broadening that significantly increases the apparent temperature when the true $T_i$ is low, independent of the degree of saturation, but once again it is possible to operate at 1–1000 times the saturation intensity (depending on $T_i$) without introducing large additional errors in the temperature measurement.

A final concern, of practical importance for the LIF measurements described in this paper, is whether saturation can distort the mean velocity measurement. When hyperfine structure is neglected and the IVDF is symmetric, it is clear from Fig. 23 that saturation has no effect on the apparent $u_i$. However, it is not immediately obvious whether this conclusion is still valid in a realistic case with an asymmetric IVDF and hyperfine structure accounted for. In Fig. 26, we show the calculated error in the mean velocity inferred from LIF as a function of $I_\nu/I_{sat}$ for Maxwellian ions with $T_i = 1$ eV and $u_i = 1$ km/s, Maxwellian ions with $T_i = 1$ eV and $u_i = 10$ km/s, and an actual asymmetric IVDF from Fig. 7(c) (where for simplicity we have assumed that the data represents the true IVDF, and then calculated the distortion of this profile that would result from hyperfine structure and saturation). The error in mean velocity is weakly dependent on the degree of saturation, but it is less than 150 m/s for all cases considered. Since this is much smaller than most of the mean velocities measured in this paper, we consider the distortion to be negligible and have not applied any correction to the data. This conclusion agrees with the results from Ref. 51, in which the mean velocity was measured as a function of laser intensity and was found to be nearly independent of $I_\nu/I_{sat}$. 

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Figure 27. Measured LIF signal as a function of laser power into the ∼2 mm diameter interrogation volume. a) $V_d = 300$ V, $z/L_{channel} = 0.57$, laser tuned to the center of the Doppler line profile. b) $V_d = 400$ V, $z/L_{channel} = 2.04$, laser tuned to the center of the Doppler line profile. c) $V_d = 400$ V, $z/L_{channel} = 2.04$, laser tuned to one of the wings of the Doppler line profile.

C. Saturation Study Data

A limited set of saturation studies were carried during the HERMeS TDU-2 LIF campaign. With the laser wavelength held fixed, the gain of the tapered amplifier was adjusted and the lock-in amplifier output level with the time constant set to 3 s was recorded. The output power as a function of tapered amplifier gain was calibrated using neutral density filters and a Thorlabs PM160 optical power meter. Three of these datasets are shown in Fig. 27. As predicted by the theory in the previous sections, the LIF signal varied nonlinearly with laser intensity for power levels at least as low as 20 mW (∼6 mW/mm²). Insufficient data was taken at lower powers to identify the intensity at which the transition to nonlinear behavior occurred. Precise agreement with the predicted saturation intensity would not have been expected in any case, since the theory does not account for a realistic nonuniform beam intensity profile.

Even though the measured signal increased nonlinearly, it did not asymptote at high laser intensities (up to $I_\nu \approx$ 60 mW/mm²). The scaling of signal intensity with power measured in the wing of the Doppler profile (Fig. 27(c)) was similar to that measured in the line center, again in agreement with the theory, which predicts that the value of $I_{sat}$ depends only on the homogeneous line width and should be independent of the location on the Doppler profile at which a measurement is taken. Note that the same laser wavelength was used for the line wing saturation studies along the "East" and "West" lines of sight, even though the IVDFs in the two directions peaked at different velocities, so the two sets of measurements were not taken at the same wavelength distance away from the peak of the Doppler profile.

D. Depletion of the Metastable Target State by a CW Laser

The discussion presented thus far is actually only half the story when it comes to LIF saturation in Hall thrusters. A continuous wave (CW) laser, or a laser that is modulated on/off at a rate that is slow compared to the transition rates for the relevant atomic processes, can deplete the metastable target state, causing the LIF signal intensity to scale nonlinearly with laser power even if $n_2(v, \nu)$ remains much less than $n_1(v, \nu)$. This may be understood by revisiting the rate equations for the populations of the lower and upper LIF states. In Eq. 20, the spontaneous de-excitation term should actually include all allowed radiative decays to lower states that can depopulate state 2; i.e., $A_{21}$ should be replaced by $\sum_j A_{2j}$. With this change, the two-level model no longer has a self-consistent steady-state solution.

If we assume for illustrative purposes that $n_2(v, \nu)$ is constant in time, then solving the modified Eq. 20 yields

$$n_2(v, \nu) = \frac{b_{12}(v, \nu)}{b_{21}(v, \nu) + \sum_j A_{2j}} n_1(v, \nu). \tag{34}$$

Plugging into Eq. 19, with no steady state assumption, we have:

$$\frac{d}{dt} (n_1(v, \nu)) = \left(-1 + \frac{b_{21}(v, \nu) + A_{21}}{b_{21}(v, \nu) + \sum_j A_{2j}}\right) b_{12}(v, \nu) n_1(v, \nu). \tag{35}$$

This equation shows that if there are multiple decay pathways for the upper LIF state (so $\sum_j A_{2j} > A_{21}$), which is true for most LIF schemes including the Xe II 834.953 nm scheme used in this paper, the population
density $n_1(v, \nu)$ of the metastable target state will decrease in time. Physically, this decrease will continue until $n_1$ is low enough that the rate of another process that populates state 1 but is not included in the two-level model (usually electron-impact excitation from the ground state or convection of metastable ions into the interrogation volume) is sufficient to balance the loss rate from laser excitations followed by spontaneous decays to other states. The threshold laser intensity for this second type of LIF saturation can be strongly dependent on location within the Hall thruster plasma. For example, when the laser beam direction is not axial, the high axial ion flow velocities in the plume ($\sim 20 \text{ km/s}$) mean that a 2 mm diameter LIF interrogation volume will be replenished with new metastable ions every $\sim 100 \text{ ns}$; thus the saturation threshold will be much higher in the plume than in the channel, where ion velocities are low and convection is not an effective re-populating mechanism. These ideas will be pursued further in a future publication.

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References


