Study of electron transport in a Hall effect thruster with 2D $r−\theta$ Particle-In-Cell simulations

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Vivien Croes,* Antoine Tavant†, Romain Lucken‡
and
Trevor Lafleur§, Anne Bourdon¶, and Pascal Chabert∥

Laboratoire de Physique des Plasmas, CNRS, Sorbonne Universités,
UPMC Universités Paris 6, Université Paris Sud, Université Paris Saclay,
École polytechnique, F-91128 Palaiseau Cedex, France

In this work, we study the plasma discharge occurring in a Hall-effect thruster (HET) channel using a 2D particle-in-cell/Monte Carlo collision (PIC/MCC) simulation. The system is configured in order to simulate the radial-azimuthal ($r−\theta$) exit plane for large radius thrusters using a Cartesian coordinate system. Thus, the $r$ (resp. $\theta$) axis corresponds to the $(Oy)$ (resp. $(Ox)$) directions.

In this HET configuration, an imposed radial magnetic field, $B_0$, is aligned along the $(Oy)$ axis, while a constant axial electric field, $E_0$, is applied along the $(Oz)$ axis, mimicking a third dimension to the system. Thus, the $E_0 \times B_0$ direction is set along the $(Ox)$ axis, where azimuthal instabilities propagate.

The present study focuses on the impact of secondary electron emission (SEE) on the plasma discharge. Thus, two models for realistic electron-induced SEE for boron nitride (BN) walls, are implemented and compared.

As already shown by previous works, electron transport is expected to be caused primarily by the presence of an electron drift instability propagating in the $E_0 \times B_0$ direction. Consequently, the electron drift instability characteristics as well as the electron mobility in the PIC/MCC simulations were quantified. Impact of SEE on the electron drift characteristics was observed not to be significant.

Moreover the electron drift instability characteristics, as well as the electron mobility estimates, from PIC/MCC simulations were compared to a recent kinetic theory, showing good agreement.

SEE are observed to play a role in a near-wall enhanced electron mobility, however, it is concluded that they play an auxiliary role in the overall electron mobility, in comparison to the electron drift instability contribution.

*Ph.D. student, Safran Aircraft Engines, vivien.croes@lpp.polytechnique.fr.
†Ph.D. student, Safran Aircraft Engines, antoine.tavant@lpp.polytechnique.fr.
‡Ph.D. student, Université Paris Saclay, romain.lucken@lpp.polytechnique.fr.
§Postdoctoral research fellow, Centre National d’Étude Spatiales, trevor.lafleur@lpp.polytechnique.fr.
¶Research director, Centre National de la Recherche Scientifique, anne.bourdon@lpp.polytechnique.fr.
∥Research director, Centre National de la Recherche Scientifique, pascal.chabert@lpp.polytechnique.fr
Nomenclature

BN = boron nitride
CIC = cloud-in-cell
CPU = computational power unit
EP = electric propulsion
GIT = gridded ion thruster
HDF5 = hierarchical data format v.5
HET = Hall effect thruster
LPPic2D = 2D-3V PIC/MCC code used to obtained the results presented in this work
MCC = Monte Carlo collision
MPI = message passing interface
PIC = particle-in-cell
PPU = power processing unit
SEE = secondary electron emission

$\Delta t$ = time-step
$\Delta x = \Delta y = \Delta z$ = cell size
$n_0$ = plasma density
$T_e$ = electron temperature
$T_i$ = ion temperature
$P_n$ = neutral gas pressure
$T_n$ = neutral gas temperature
$N$ = number of particles into the simulation
$NG$ = number of grid-cells
$N_A$ = number of time-step over which the output values are averaged
$L_z$ = system length along the ($Oz$) axis
$E_0$ = electric field arbitrarily imposed along the ($Oz$) axis
$B_0$ = magnetic field arbitrarily imposed along the ($Oy$) axis
$\sigma$ = re-emission probability
$\sigma_0$ = probability of attachment
$\epsilon^*$ = cross-over energy
$\sigma_{\text{max}}$ = maximum re-emission probability
$w_0$ = emission threshold energy
$k_s$ = smoothness factor of the surface
$w_{\text{max}}$ = primary electron energy
$\gamma_{\text{max}}$ = emission coefficient at the maximum of emission for normal incidence
$L_{\text{die}}$ = dielectric thickness
$\epsilon_{\text{die}}^r$ = $\epsilon_{\text{die}}/\epsilon_0$, dielectric relative permittivity
$\lambda$ = electron drift instability wavelength
$f$ = electron drift instability frequency
$v_{\text{ph}}$ = electron drift instability phase velocity
$g_{\text{max}}$ = electron drift instability growth rate
$\tau_g$ = electron drift instability growth time
$\mu_{\text{pic}}$ = electron mobility measured from the PIC simulations
$\mu_{\text{cla}}$ = electron mobility from classical diffusion theory
$\mu_{\text{eff}}$ = electron mobility from kinetic theory
$\mu_{\text{sat eff}}$ = electron mobility at saturation from kinetic theory
$\bar{\sigma}$ = SEE yield averaged in time and over the electron population
I. Introduction

Plasma thrusters are nowadays a well-known technology. Indeed, the first ion engine was launched in 1964, while the first Hall effect thruster (HET) space flight took place in 1974. Yet, most of these electric propulsion (EP) technologies are complex systems, and their operation is still poorly understood. Numerous EP technologies exist, however as HETs are one of the most promising systems, the present work will focus on the simulation of a HET.

As illustrated in Figure 1, typical HETs consist of three main parts, listed below:

**Annular ceramic channel** where the propellant gas is injected (through a porous anode), ionized, accelerated, and ejected through one end. This channel usually has a length of the order of centimeters. In addition, densities in the channel are typically in the range between $10^{17}$ to $10^{18}$ m$^{-3}$ for the plasma, and $10^{18}$ to $10^{20}$ m$^{-3}$ for the neutral gas.

**External hollow cathode** (asymmetrically located near one side of the channel, or along its central axis), providing electrons to sustain the plasma discharge inside the channel, as well as neutralizing the exiting ion beam. A large potential difference (100's of Volts) is applied between the anode and cathode, which accelerates the ions to high velocities to generate thrust. This potential difference also causes some electrons from the cathode to travel upstream inside the channel towards the anode.

**Magnetic circuit** specifically designed and used to impose a predominantly radial magnetic field (10's of mT) in the channel region. This magnetic field acts to impede electrons, and increases their residence time in the channel, so as to allow a higher probability of ionization, and thus ensure maximal use of the injected propellant gas. HETs are considered electrostatic devices, since this external magnetic field imposed by the coils is much more important than that produced by any fluctuations generated by charged particles motion. The generated ions are then accelerated by the electrostatic fields imposed by the applied voltage on the anode and cathode.

These three main components are fixed together through a metallic structure allowing electrostatic as well as thermal control of the thruster. This structure is then fixed to the spacecraft, often through an extensible arm system, and connected to a power processing unit (PPU) and a propellant tank. Although these parts are needed for the thruster operation, the present work will focus on the channel, and the plasma discharge occurring in it.

Numerous studies have shown that the electron mobility across the magnetic field tends to be anomalously high in comparison to the mobility predicted by classical diffusion theories based on standard electron-neutral or electron-ion collisions, in particular near the thruster exit and in the near-plume region. In order to explain this anomaly, four different mechanisms have been proposed:

**Intense SEE** arising from electron-wall collisions enhancing the electron mobility near the channel walls,

**Sheath instabilities** in the radial direction due to intense secondary electron emission,

**Strong azimuthal instabilities** due to large electron drift velocities in the azimuthal direction,

**Fluid instabilities** driven by strong gradients in the plasma discharge.

Moreover the role of thruster wall materials on this anomalous transport has been experimentally highlighted in numerous studies. However evidence suggests that electron-wall collisions and SEE are not sufficient to explain the observed cross-field electron transport. Thus, the most plausible mechanism seems to be the formation of instabilities in the azimuthal direction, as highlighted by both experimental and numerical studies. These instabilities feature large amplitude oscillations in both the plasma density and electric field. They have frequencies in the MHz range, wavelengths of the order of the mm, and electric field amplitudes almost as large as the axial accelerating field itself.

These azimuthal instabilities were observed by 2D $z - \theta$ PIC simulations and 2D $r - \theta$ PIC simulations, investigating the electron drift instability, suggesting this instability as the main phenomenon causing the anomalous transport. Complementing these 2D studies, 1D PIC simulations have obtained some insights into this electron drift instability. Generalizing methods used in these 1D PIC simulations, a
simplified 2D $r - \theta$ model was developed.\textsuperscript{12} This model allowed us to investigate the importance of 2D effects, and to compare the PIC simulations results with a recent kinetic theory.\textsuperscript{32}

However our first $r - \theta$ model,\textsuperscript{12} while giving us valuable insights about the plasma discharge behavior, included some oversimplifications. Indeed, SEE processes, as well as dielectric walls, were not modeled. Consequently, in this work, the model has been extended in order to realistically take into account the impact of the electron emissions from the ceramic walls on the plasma discharge.

The present work firstly presents the model used in the simulations, then some important results from the kinetic theory\textsuperscript{12,31,32} are reviewed, and finally two SEE models set-up for dielectric walls are compared.

\section*{II. Model description}

Results presented in this work were obtained using an independently developed 2D-3V PIC code: \textit{LPPic2D}.\textsuperscript{12,13} This code does not use any geometric or parametric scaling factors, and correctly preserves all spatial and temporal scales.

\subsection*{A. Particle-In-Cell/Monte Carlo collisions (PIC/MCC) simulations}

\textit{LPPic2D} uses the classical structure of a 2D-3V PIC/MCC code.\textsuperscript{4} It exploits a fixed Cartesian structured mesh, with square cells. The time-step ($\Delta t$), as well as the cell size ($\Delta x = \Delta y$), are parameters, and chosen so as to resolve the electron plasma frequency, and to satisfy the Courant-Friedrichs-Lewy (CFL) condition.

Particles are initialized at a given plasma density, $n_0$, with a given temperature ($T_e$ for electrons, $T_i$ for ions). The number of particles initialized in the system, $N$, is a parameter chosen in order to obtain a proper number of particles/grid-cell ratio ($N/NG$).

The code uses the leapfrog scheme for both charged species (ions and electrons) to explicitly integrate the equations of motion.\textsuperscript{4} The Boris scheme\textsuperscript{7} is used for electrons in order to efficiently model their trajectories into the magnetic field. Neutrals are treated as a fixed, homogeneous, background at a temperature, $T_n$, and pressure, $P_n$.

Collisional processes between charged particles and the neutral background are modeled using a Monte Carlo collisions (MCC) algorithm\textsuperscript{51} for a xenon plasma (131.3 AMU, with collisional cross-sections from\textsuperscript{40}). Ionization processes, as well as the three excitation levels and elastic collision processes, are modeled for
electron/neutral collisions. For ion/neutral collisions, charge-exchange and elastic collision processes are modeled.

Interpolation of particle densities at the grid-points, as well as the electric fields at the particle position, are obtained through a linear cloud-in-cell (CIC) scheme. Since the simulation is electrostatic, the electric field is obtained from the plasma potential by solving Poisson’s equation at the grid-points.

Finally, diagnostics are used to store relevant simulation outputs. In order to reduce the level of statistical noise in the simulations, results are averaged over a fixed number of time-steps, \( N_A \) (usually \( N_A = 2000 \), as given in Table 1).

Because of the nature of 2D PIC simulations of HETs, \( LPPic2D \) requires a significant amount of computational power. Consequently, it has been chosen to take advantage of the available modern computational methods thanks to the parallelization of the code through a spatial domain decomposition using the MPI library. The solving of Poisson’s equation is executed thanks to the HYPRE library, while the output files are encoded using the HDF5 library.

B. Hall-effect thruster (HET) model

\( LPPic2D \) has been used to model a system close to the \(( r – \theta )\) plane near the exit of a HET discharge channel.

1. Large radius HET model

Results have already been obtained using a simplified model that does not include any curvature or SEE, and where the radial walls are metallic and grounded.

Since it has been shown not to play a significant role on the plasma discharge behavior, at least in large radius thrusters, we neglect the channel curvature in the following work. Consequently, the \( r \) (respectively \( \theta \)) axis corresponds in our set-up to the \((Oy)\) axis (respectively \((Ox)\)). For the sake of clarity, only Cartesian coordinates will be used in the following.

In this set-up, the electric field in the \((Ox-Oy)\) simulation plane is obtained from charge densities on the grid by solving Poisson’s equation. Nonetheless, particles can move along the \((Oz)\) axis, where a constant and homogeneous electric field, \( E_0 \), is arbitrarily imposed. The system length along \((Oz)\) is set to a finite value, \( L_z \), in order to mimic a 3D behavior. This method is a 2D generalization of 1D methods from, and its configuration has already been previously detailed.

Concerning the channel walls, the metallic walls are kept in order to isolate the effects of SEE and allow a clear comparison with previous results.

Typical parameters used in the following simulations are summarized in Table 1. Using this simulation set-up, a typical simulation over 10 \( \mu s \) takes \( \approx 32 \) h with 360 CPUs.

2. SEE models

Complementing this first model, SEE processes were implemented. In \( LPPic2D \), two models have been implemented to simulate electron induced SEE.

1. A commonly used linear model. In this model, the incident electron energy, \( \epsilon \), is used to estimate the re-emission probability, \( \sigma(\epsilon) \). A linear function describes this relation, as given by:

\[
\sigma(\epsilon) = \min \left( \sigma_0 + \frac{\epsilon}{\epsilon^*}[1 - \sigma_0], \sigma_{\text{max}} \right)
\]

where \( \epsilon^* \) is the crossover energy, \( \sigma_0 \) the probability of attachment, and \( \sigma_{\text{max}} \) the maximum re-emission probability. Values of \( \sigma_0 \), \( \sigma_{\text{max}} \), and \( \epsilon^* \) can be fitted to experimental values in order to model BN walls, commonly used in HETs.

2. A more complex model, where not only the electron incident energy is used, but also its incident angle, \( \theta \). This model was developed by Vaughan with the parameters corresponding to the BN dielectric walls. In this model, the re-emission probability, \( \sigma(\epsilon, \theta) \), is obtained as:

\[
\sigma(\epsilon, \theta) = \gamma_{\text{max}}(\theta) \left( v(\epsilon, \theta) \exp[1 - v(\epsilon, \theta)] \right)^k
\]
Table 1. Standard operating and numerical parameters used in the 2D PIC simulations of a HET channel.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas</td>
<td></td>
<td>Xe</td>
</tr>
<tr>
<td>$L_x$</td>
<td>[cm]</td>
<td>0.5</td>
</tr>
<tr>
<td>$L_y$</td>
<td>[cm]</td>
<td>2.0</td>
</tr>
<tr>
<td>$L_z$</td>
<td>[cm]</td>
<td>1.0</td>
</tr>
<tr>
<td>$B_0$</td>
<td>[G]</td>
<td>200</td>
</tr>
<tr>
<td>$E_0$</td>
<td>[Vm$^{-1}$]</td>
<td>$2 \times 10^4$</td>
</tr>
<tr>
<td>$n_0$</td>
<td>[m$^{-3}$]</td>
<td>$3 \times 10^{17}$</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>[s]</td>
<td>$4 \times 10^{-12}$</td>
</tr>
<tr>
<td>$\Delta x = \Delta y = \Delta z$</td>
<td>[cm]</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$T_e$</td>
<td>[eV]</td>
<td>5.0</td>
</tr>
<tr>
<td>$T_i$</td>
<td>[eV]</td>
<td>0.1</td>
</tr>
<tr>
<td>$T_{see}$</td>
<td>[eV]</td>
<td>1.0</td>
</tr>
<tr>
<td>$N$</td>
<td>[particles]</td>
<td>$25 \times 10^6$</td>
</tr>
<tr>
<td>$NG$</td>
<td>[grid-points]</td>
<td>$255 \times 1000$</td>
</tr>
<tr>
<td>$N/NG$</td>
<td>[part/cell]</td>
<td>$\approx 100$</td>
</tr>
<tr>
<td>$P_n$</td>
<td>[mTorr]</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_n$</td>
<td>[K]</td>
<td>300</td>
</tr>
<tr>
<td>$n_g$</td>
<td>[m$^{-3}$]</td>
<td>$3.22 \times 10^{19}$</td>
</tr>
</tbody>
</table>

where

$$v(\epsilon, \theta) = \frac{\epsilon - w_0}{w_{\max}(\theta) - w_0}$$  \hspace{1cm} (3)

$$w_{\max} = w_{\max,0} \left(1 + \frac{k_s \theta^2}{\pi}\right)$$  \hspace{1cm} (4)

$$\gamma_{\max}(\theta) = \sigma_{\max} \left(1 + \frac{k_s \theta^2}{\pi}\right)$$  \hspace{1cm} (5)

$$k = \begin{cases} 
0.62 & \text{if } \epsilon < w_{\max}(\theta) \\
0.25 & \text{if } \epsilon > w_{\max}(\theta) 
\end{cases}$$  \hspace{1cm} (6)

with $w_0$ the emission threshold energy, $k_s$ the smoothness factor of the surface (the lower the rougher), and $w_{\max}$ and $\gamma_{\max}$ are respectively the primary electron energy and the emission coefficient at the maximum of emission for normal incidence.

Parameters needed to set-up the SEE models used in the following simulations are summarized in Table 2. As detailed in Equations 1 and 5, the value $\sigma_{\max}$ is shared by the two models, and obtained from experimental studies.\textsuperscript{14} This is illustrated in Figure 2, where these the SEE yield is plotted as a function of the electron energy for both models. For each simulation, one can chose which model is used.
Figure 2. SEE yield as a function of the electron incident energy, $\epsilon$: (green) for the linear model, and (blue) the Vaughan model for an electron with a normal incident angle (i.e. $\theta = 0$).

Table 2. Parameters used to configure the SEE models for BN ceramic.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$\epsilon^*$</td>
<td>[eV]</td>
<td>35.04</td>
</tr>
<tr>
<td></td>
<td>$\sigma_0$</td>
<td></td>
<td>0.578</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{max}}$</td>
<td></td>
<td>2.9</td>
</tr>
<tr>
<td>Vaughan</td>
<td>$k_s$</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>$w_0$</td>
<td>[eV]</td>
<td>13.0</td>
</tr>
<tr>
<td></td>
<td>$w_{\text{max,0}}$</td>
<td>[eV]</td>
<td>500.0</td>
</tr>
</tbody>
</table>

III. Kinetic theory considerations

In order to verify a kinetic theory through comparisons with the results from the PIC simulations, it is necessary to review some of its main results.

A. Electron drift instability characteristics

Features of the electron drift instability such as the wavelength, frequency, or phase velocity are analytically predicted by the kinetic theory. Corresponding analytical expressions are presented below.

The electron drift instability wavelength, $\lambda$, is given by:

$$\lambda = 2\pi \lambda_{D_e} \sqrt{2}$$  \hspace{1cm} (7)

where $\lambda_{D_e} = \sqrt{\frac{\epsilon_0 T_e}{|q| n}}$ is the Debye length.

Furthermore we obtain its frequency, $f$, from:

$$f = \frac{\omega_{\text{pi}}}{2\pi \sqrt{3}}$$  \hspace{1cm} (8)
with \( \omega_{pi} = \sqrt{nq^2/(\epsilon_0 m_i)} \) the ion plasma frequency.

Finally the phase velocity, \( v_{ph} \), is given by:

\[
v_{ph} = \sqrt{2/3} c_s
\]

where \( c_s = \sqrt{|q| T_e/m_i} \) is the ion sound speed.

**B. Electron enhanced transport**

The mean electron cross-field mobility, \( \mu_{\text{pic}} \), is obtained from the simulations as:

\[
\mu_{\text{pic}} = \frac{\sum_{j=1}^{N_e} v_{jz}}{N_e E_0}
\]

where the summation is over all electrons in the simulation domain, \( N_e \), and \( v_{jz} \) is the speed along \((Oz)\) of the \( j^{th} \) electron.

Classical diffusion theory gives us an estimate of the electron anomalous transport, \( \mu_{\text{cla}} \):

\[
\mu_{\text{cla}} = \frac{|q|}{m_e \nu_m} \left( 1 + \frac{\omega_{ce}^2}{\nu_m^2} \right)
\]

with \( \omega_{ce} = |q| B_0/m_e \) the electron cyclotron frequency, and \( \nu_m \) the electron-neutral collision frequency, which is measured from the simulation, as:

\[
\nu_m(t') = \int_{t'}^{t'+T} \frac{dt}{T} \frac{N_{\text{collisions}}(t)}{N_{\text{electrons}}(t)}
\]

where \( N_{\text{collisions}}(t) \) is the number of electron-neutral collisions at each time-step, \( N_{\text{electrons}}(t) \) is the number of electrons in the system at each time-step, and \( T = 2 \times 10^3 \Delta t = 8\text{ns} = N_A \Delta t \) an averaging period used in order to minimize the statistical noise levels due to the PIC model, with \( N_A \) given in Table 1.

Finally, in the case where the electron drift instability forms, the cross-field electron mobility is described by:

\[
\mu_{\text{eff}} = \frac{|q|}{m_e \nu_m} \left[ 1 - \frac{\omega_{ce}^2}{\nu_m^2} \frac{\langle n_e E_z \rangle}{n_e E_0} \right]
\]

where \( n_e \) is the electron density, \( E_0 \) the norm of the axial electric field, and \( E_z \) is the electric field in the \((Ox)\) direction. As will see in the following, the term in angled brackets is generally negative so that \( \langle n_e E_z \rangle < 0 \), and thus fluctuations enhance the electron mobility. We follow in time from the simulation the value of the correlation term \( \langle n_e E_z \rangle \) using:

\[
\langle n_e E_z \rangle(t') = \int_{0}^{L_z} dx \int_{0}^{L_y} dy \int_{0}^{L_y} dt' \frac{T}{T} n_e(x, y, t) E_z(x, y, t)
\]

where the integrals are evaluated numerically by using the electron density, \( n_e(x, y, t) \), and electric field along \((Ox)\), \( E_z(x, y, t) \), given by the simulation at each spatial grid point and at each time-step. The value is then averaged over \( N_A \) time-steps, corresponding to the \( T \) period in Equation 14.

At saturation, and in our simulation set-up, Equation 13 can be estimated as:

\[
\mu_{\text{eff}}^{\text{sat}} = \frac{|q|}{m_e \nu_m} \left[ 1 + \frac{|q|}{m_e \nu_m} \frac{B_0}{E_0} \frac{v_{zi} T_e}{4\sqrt{6} c_s L_z} \right]
\]

where \( v_{zi} \) is the ion exit velocity along \((Oz)\), which can easily be estimated by neglecting ion/neutral collisions and the initial ion temperature, as:

\[
v_{zi} = \sqrt{2 q E_z L_z / m_i}
\]
Since it has been properly shown\(^1\) through empirical studies, a Bohm-like electron mobility model (e.g. proportional to the inverse of the magnetic field, \(B_0\), or squared magnetic field) is not appropriate to model electron cross-field transport in a HET channel. Consequently, Bohm-like mobility models are not taken into account in this work.

IV. Impact of secondary electron emission (SEE)

We now evaluate the effects of the SEE models detailed in Section II. For the sake of clarity, the simulation using the linear SEE model and no dielectrics will be referred to as: \textit{linear}, and the one using the Vaughan SEE model and no dielectrics will be referred to as: \textit{Vaughan}. The case without SEE is referred to as \textit{noSEE} in the following.

A. Electron drift instability features

In both simulations, the electron drift instability is observed to grow and saturate in \(\approx 1 - 2 \mu s\), as we can observe in Figures 3 and 4. Figures 3 and 4 feature a transitional phase during the first \(2 \mu s\), where some large fluctuations are observed. However, after this initialization phase, the electron drift instability is observed to saturate. Consequently, the impact of the SEE on the electron drift instability features is considered not significant for both cases, as summarized in Table 3.

Table 3. Comparison between physical values measured from the simulation and predictions from the kinetic theory about the instability characteristics for the two SEE models, and with \(L_{\text{diel}} = 0\) (i.e. with grounded metallic walls).

<table>
<thead>
<tr>
<th>Measured values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>linear</td>
</tr>
<tr>
<td>Vaughan</td>
</tr>
<tr>
<td>noSEE</td>
</tr>
</tbody>
</table>

| Case | \(|\delta n_e|/n_e|\) [%] | \(|\delta \Phi|/T_e|\) [%] |
|------|-----------------|----------------|
| linear | 20            | 36             |
| Vaughan | 18            | 40             |
| noSEE    | 19            | 54             |

<table>
<thead>
<tr>
<th>Analytical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>(Equation 7)</td>
</tr>
<tr>
<td>-</td>
</tr>
</tbody>
</table>

| Case | \(|\delta n_e|/n_e|\) [%] | \(|\delta \Phi|/T_e|\) [%] |
|------|-----------------|----------------|
| - | 33             | 33             |

All values presented in Table 3 are averaged during the last \(5 \mu s\) of the simulation, when the instability is saturated, in order to lower the noise level. Moreover, the uncertainty on the \(T_e\) measurement (which is obtained from the measurement of \(\epsilon_e\)) is about \(\approx \pm 5 \text{[eV]}\). This uncertainty is then echoed in the subsequent estimates, which give the analytical values presented in Table 3.

Measurements of frequency and wavelength were done using Fourier transforms. The error margin is estimated as \(\approx \pm 0.5 \text{[MHz]}\) for the frequency, and \(\approx \pm 0.1 \text{[mm]}\) for the wavelength.

Nevertheless, concerning the estimation of the phase velocity, \(v_{ph}\), from the PIC simulation, the method used consists of measuring the correlation function between each 2D spatial plot of the plasma potential (a plot is taken every \(N_A\) time-step, i.e. every \(N_A \Delta t = 8\text{ns}\), then the mean value of the correlation is used in order to estimate the phase velocity. This method allows for an error margin estimated to be approximately

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\(^1\) The 35th International Electric Propulsion Conference, Georgia Institute of Technology, USA
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±0.5 \[10^3 \text{ms}^{-1}\].

The oscillation amplitudes are measured directly from the time evolution plots from 1D cuts in the 2D domain. An example of such a plot is shown in Figure 3. Consequently, the error margin is \(\approx 10\%\).

Table 3 shows that with the error margin taken into account we approximately have \(\lambda f \approx v_{ph}\), as it could be expected from Equations 7, 8, and 9. This shows that, despite the error margins, the PIC/MCC values and analytical estimates give satisfactory results, confirming the kinetic theory.\(^{32}\)

However, as highlighted in Figures 3 and 4, the plasma potential is impacted by the presence of SEE processes in the simulation, and is lower than in the noSEE case.\(^{12}\) Especially in the case of the linear SEE model, where the plasma potential is significantly lower than the case without SEE.\(^{12}\)

### B. Electron anomalous cross-field transport

For both simulations, the electron anomalous transport is measured, and compared to analytical values, as summarized in Table 4.

The averaging of physical values needed to estimate the mobilities presented in Table 4 leads us to consider an error margin of \(\approx \pm 0.5 \text{[m}^2\text{V}^{-1}\text{s}^{-1}]\) concerning the PIC values of mobilities, while analytical values have an error margin of \(\approx \pm 0.1 \text{[m}^2\text{V}^{-1}\text{s}^{-1}]\). Furthermore, as for other values measured from the PIC simulations, the mobilities are averaged during the last 5\(\mu\)s of the simulation, when the instability is saturated, in order to lower the noise level.

Values of electron mobility averaged in time and over the simulation domain show a lower value in the cases with SEE processes included. Consequently, in order to deepen the investigation over the electron cross-field mobility, \(\mu_{\text{pic}}\) is studied spatially, by plotting its value from on wall to another, as illustrated in Figure 5.

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**Figure 3.** Time evolution of the plasma potential (in [V]) profile taken from 1D cuts of the 2D \((Ox - Oy)\) simulation domain taken along \((Oy)\), at \(x = L_x/2\). Simulation done with parameters from Table 1 and 2 with the linear SEE model.
Figure 4. Time evolution of the plasma potential (in \[\text{V}\]) profile taken from 1D cuts of the 2D \((Ox - Oy)\) simulation domain taken along \((Oy)\), at \(x = L_x/2\). Simulation done with parameters from Tables 1 and 2 with the Vaughan SEE model.
Table 4. Comparison between physical values measured from the simulation and predictions from the kinetic theory for the electron cross-field mobility.

<table>
<thead>
<tr>
<th>Case</th>
<th>Measured values</th>
<th>Analytical values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mu_{\text{pic}} )</td>
<td>( \mu_{\text{eff}} )</td>
</tr>
<tr>
<td>(Equation)</td>
<td>(10)</td>
<td>(13)</td>
</tr>
<tr>
<td>linear</td>
<td>5.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Vaughan</td>
<td>5.6</td>
<td>4.9</td>
</tr>
<tr>
<td>noSEE</td>
<td>6.7</td>
<td>5.8</td>
</tr>
</tbody>
</table>

Figure 5. Radial profile (along \((Oy)\), from one wall to another) of the electron mobility, \( \mu_{\text{pic}} \) (Equation 10), averaged in time and along \((Ox)\): (blue) from the simulation with the linear SEE model,\(^{43}\) (red) the simulation using the Vaughan model,\(^{52}\) and (black) the case without SEE.

Figure 5 highlights the fact that the mobility in the center of the simulation does not vary significantly when SEE processes are modeled, with only the linear case showing a slightly lower mobility in this region. Moreover, the near-wall mobility already observed in simulations\(^{27,37,48}\) is here well observed, especially in the linear case.

C. SEE yield

As observed previously, the linear case shows a significantly more important difference with the Me case than the Vaughan one. As illustrated by Figure 6, it appears the SEE yield of the linear case saturates at a much higher value than the Vaughan case.

Indeed despite a “burst” in the simulation beginning, the Vaughan case saturates at a value of \( \bar{\sigma} \approx 0.65 \), while the linear case is observed to saturate at \( \bar{\sigma} \approx 0.99 \).

In HETs, electrons have been observed to hit the walls with a mainly normal incidence angle.\(^{24}\) This can be verified by running a simulation using the Vaughan model, but where every electron is considered to have
a normal incidence angle. Both simulations are observed to provide the exact same results. Thus it appears the incidence angle of the electron does not seem to play a major role in the SEE process, at least in large radius thrusters.

V. Discussion and conclusion

The present work has highlighted that the use of realistic electron-induced SEE models has, despite not significantly impacting the plasma discharge behavior and the electron mobility, shown two different behaviors. While on the one hand, the Vaughan model shows a low value of the SEE yield at saturation, on the other hand, the linear model is characterized by a very high value of the SEE yield at saturation with $\bar{\sigma} \approx 0.99$. This asks the question of reliable electron-induced SEE models for BN walls, although Monte Carlo based SEE models were not tested in this work.

The 2D-3V PIC/MCC model used in the present work presents a not so obvious limitation. Since the Poisson’s equation is only solved in the $(Ox - Oy)$ plane, but not along the $(Oz)$ direction, so the wavenumber of any fluctuations along $(Oz)$ is zero. This implies that convection of the instability away from the simulation plane is not correctly modeled. We have tried to account for this by using a finite axial length, and by removing particles which cross the boundaries in this direction. But this still only represents an approximation, where the $(Oz)$ direction is mimicked by the model, since the wave propagation along $(Oz)$ cannot be properly modeled without solving Poisson’s equation along this axis. Unfortunately the solution to this limitation would be to fully kinetically model a 3D system, which requires tremendous computational resources even when geometrical scaling is used.

The present work has allowed us to highlight the complimentary role of SEE and the electron drift instability on the electron cross-field mobility. In the linear case, where the SEE is intense, the near-wall mobility has been clearly identified. However, since the mean value of the electron cross-field mobility is not observed to vary significantly with the SEE intensity, it seems that the electron drift instability and the SEE processes affect in a coupled manner the electron mobility. Nevertheless, in both cases, linear and Vaughan, the near-wall conductivity is observed to only play a minor role in the electron cross-field mobility,
in comparison to the contribution of the electron drift instability.

Moreover, the results presented in this work confirm the kinetic theory developed to describe the electron drift instability and its effects on the electron mobility. Complementing previous results, where this kinetic theory was compared to PIC/MCC results, this work shows good agreement in a more realistic case, including SEE models.

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